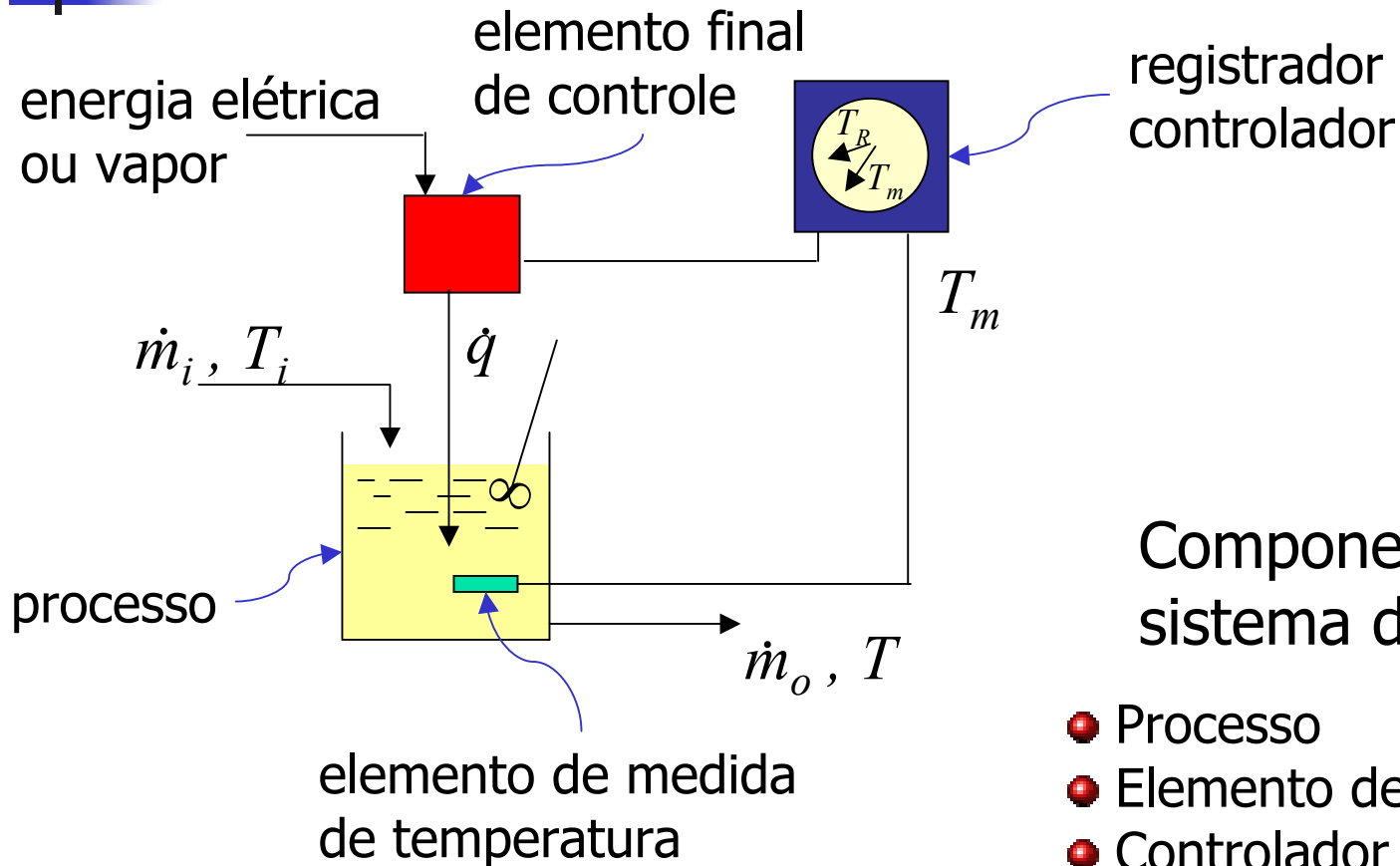


SISTEMAS LINEARES EM MALHA FECHADA



O sistema de controle

Sistema de controle para um tanque de aquecimento

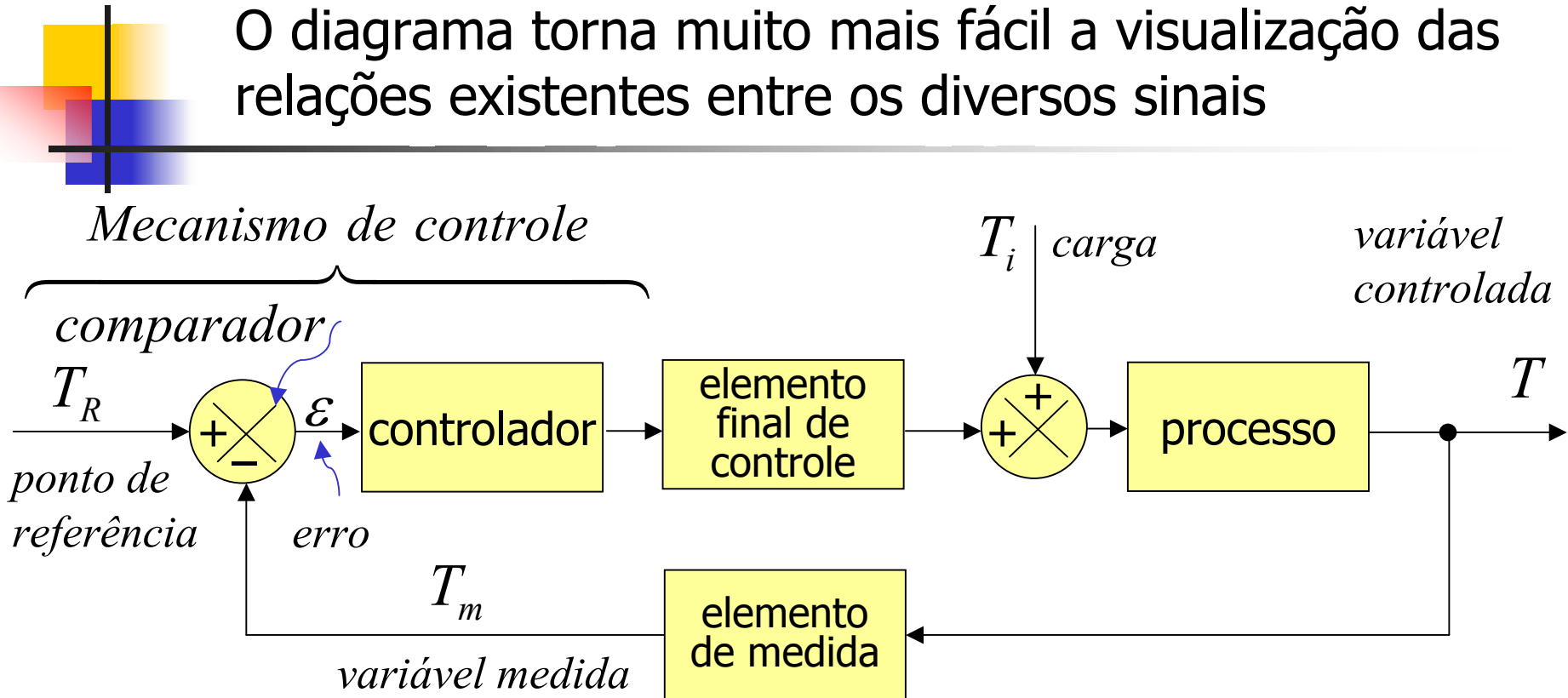


Componentes de um sistema de controle:

- Processo
- Elemento de medida
- Controlador
- Elemento final de controle

Diagrama de Blocos

O diagrama torna muito mais fácil a visualização das relações existentes entre os diversos sinais



Realimentação negativa x

Realimentação positiva

Problema servo x

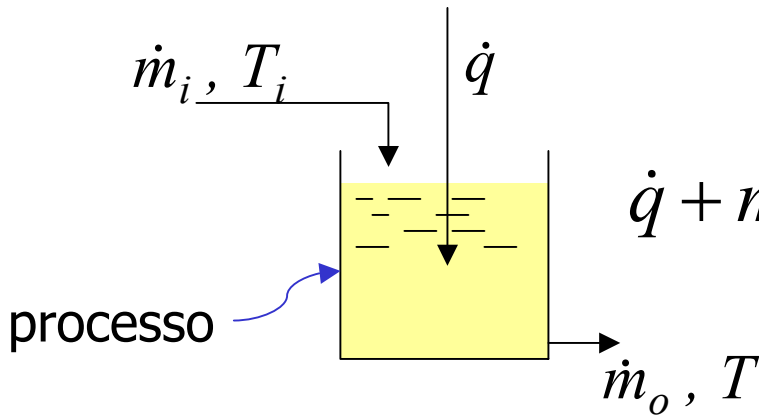
Problema regulador

Desenvolvimento do Diagrama de Blocos

Processo

Vazão constante $\rightarrow \dot{m}_i = \dot{m}_o = \dot{m}$

Conservação da energia:



$$\dot{q} + \dot{m}C(T_i - T_0) - \dot{m}C(T - T_0) = \rho CV \frac{dT}{dt}$$

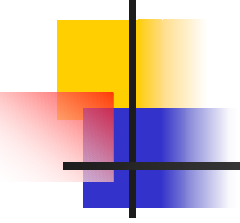
$T_0 =$ Temperatura de referência da entalpia

Em regime permanente: $\dot{q}_s + \dot{m}C(T_{i_s} - T_0) - \dot{m}C(T_s - T_0) = 0$

Subtraindo:

$$\dot{q} - \dot{q}_s + \dot{m}C[(T_i - T_{i_s}) - (T - T_s)] = \rho CV \frac{d(T - T_s)}{dt}$$

VARIÁVEIS DESVIO


$$\hat{T}_i = T_i - T_{i_s}$$

$$\hat{T} = T - T_s$$

$$\dot{Q} = \dot{q} - \dot{q}_s$$


$$\dot{q} - \dot{q}_s + \dot{m}C[(T_i - T_{i_s}) - (T - T_s)] = \rho CV \frac{d(T - T_s)}{dt}$$

$$\dot{Q} + \dot{m}C(\hat{T}_i - \hat{T}) = \rho CV \frac{d\hat{T}}{dt} \quad \Rightarrow \quad \frac{\dot{Q}}{\dot{m}C} + \hat{T}_i - \hat{T} = \frac{\rho V}{\dot{m}} \frac{d\hat{T}}{dt}$$

Aplicando a transformada de Laplace com $\tau = \frac{\rho V}{\dot{m}}$

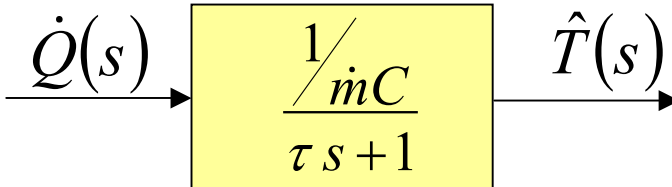
$$\frac{\dot{Q}(s)}{\dot{m}C} + \hat{T}_i(s) - \hat{T}(s) = \tau s \hat{T}(s)$$

$$\hat{T}(s)(\tau s + 1) = \frac{\dot{Q}(s)}{\dot{m}C} + \hat{T}_i(s)$$

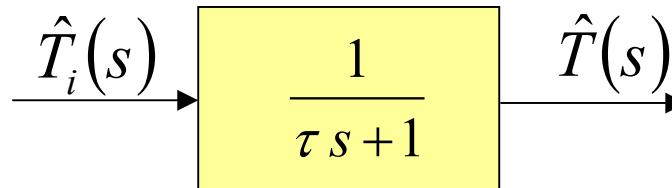

$$\hat{T}(s)(\tau s + 1) = \frac{\dot{Q}(s)}{\dot{m}C} + \hat{T}_i(s)$$

$$\hat{T}(s) = \frac{1/\dot{m}C}{\tau s + 1} \dot{Q}(s) + \frac{1}{\tau s + 1} \hat{T}_i(s)$$

Se houver variação apenas em $\dot{Q}(s)$, então $\hat{T}_i(s) = 0$
e a função de transferência que relaciona $\hat{T}(s)$ a $\dot{Q}(s)$ é:

$$\frac{\hat{T}(s)}{\dot{Q}(s)} = \frac{1/\dot{m}C}{\tau s + 1}$$


Se houver variação apenas em $\hat{T}_i(s)$, então $\dot{Q}(s) = 0$
e a função de transferência que relaciona $\hat{T}(s)$ a $\hat{T}_i(s)$ é:

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{1}{\tau s + 1}$$


$$\hat{T}(s) = \frac{1/\dot{m}C}{\tau s + 1} \dot{Q}(s) + \frac{1}{\tau s + 1} \hat{T}_i(s)$$

Esta equação é representada no diagrama de blocos

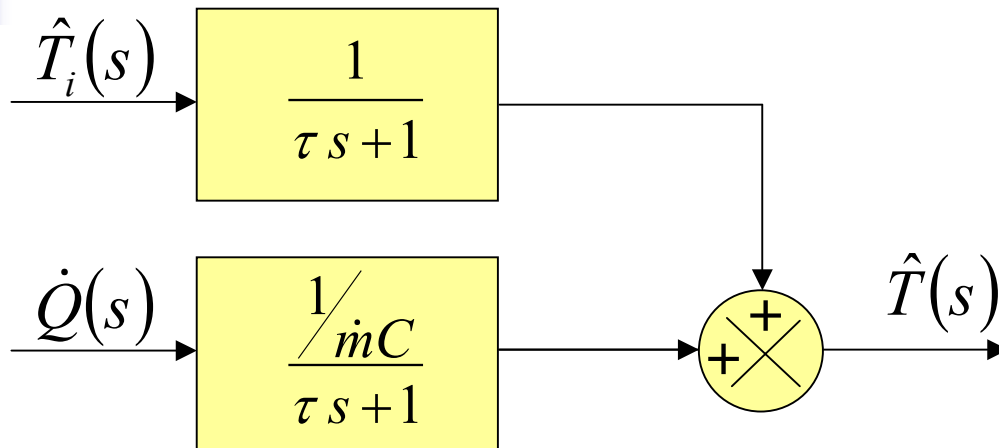
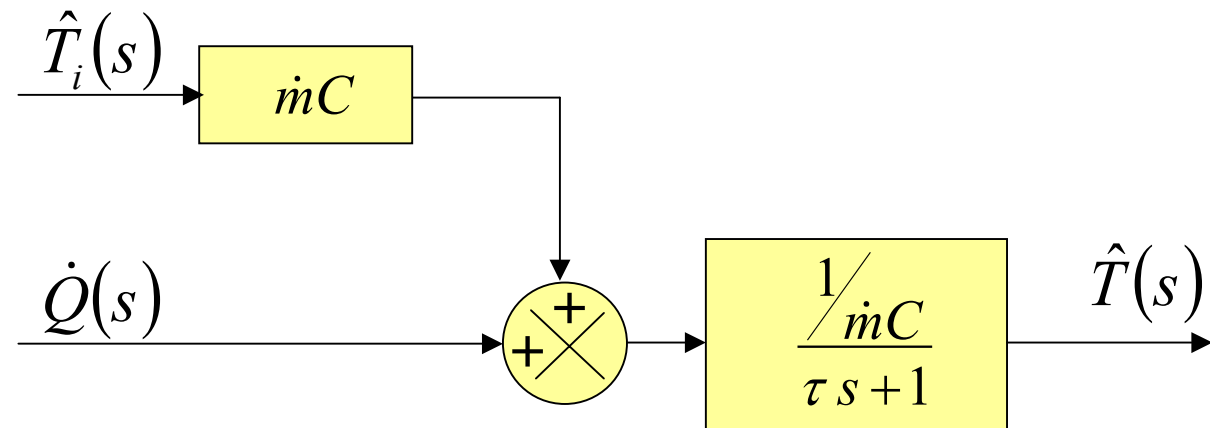
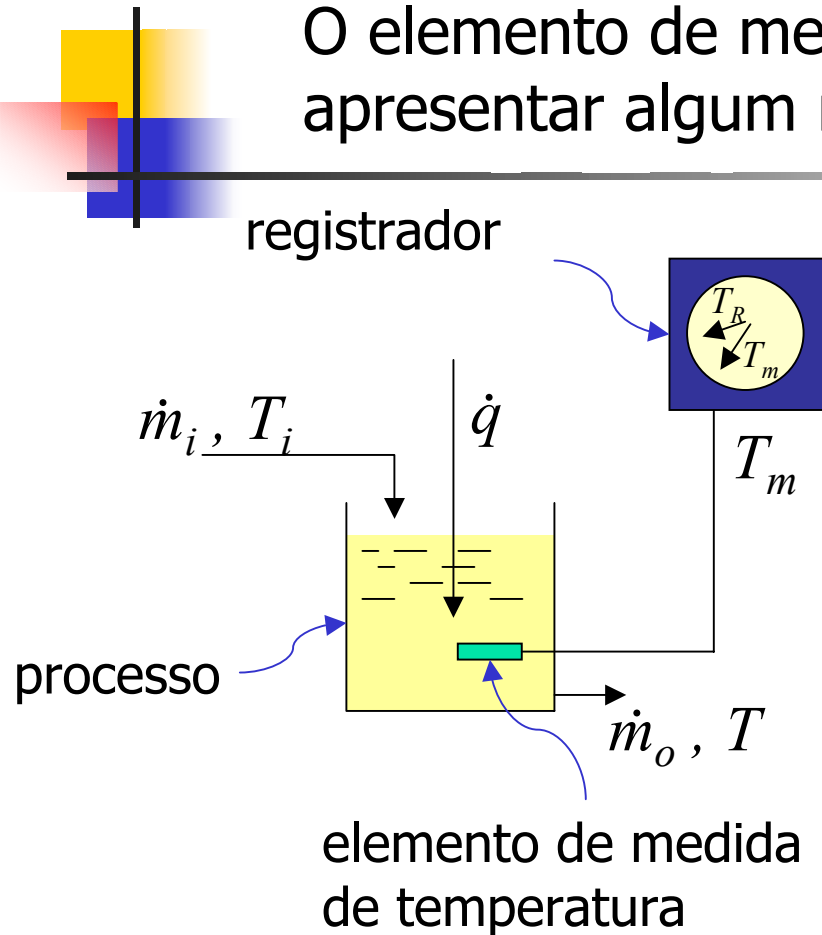


Diagrama de blocos equivalente



Elemento de Medida

O elemento de medida de temperatura pode apresentar algum retardo por transporte



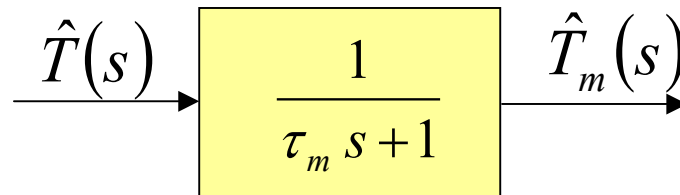
$$\frac{\hat{T}_m(s)}{\hat{T}(s)} = \frac{1}{\tau_m s + 1}$$

onde:

τ_m = constante de tempo do termômetro

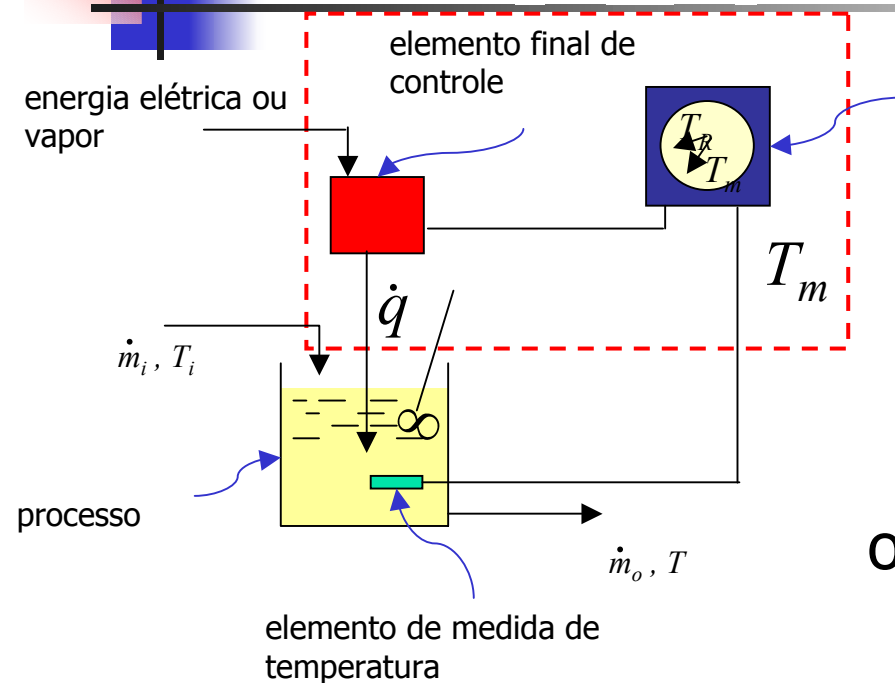
$$\hat{T}_m = T_m - T_{m_s}$$

$$\hat{T} = T - T_s$$



Controlador e Elemento Final de Controle

Por conveniência, os blocos que representam o controlador e o elemento final de controle são combinados em um só bloco



O controlador será considerado do tipo **PROPORCIONAL**

$$\dot{q} = K_c \varepsilon + A$$

onde: $\varepsilon = \hat{T}_R - \hat{T}_m$

$$\hat{T}_R = T_R - T_{R_s}$$

\hat{T}_{R_s} = temperatura de referência "normal"

K_c = sensibilidade proporcional ou ganho do controlador

A = fluxo de calor quando $\varepsilon = 0$

O uso da variável \hat{T}_R permite-nos considerar os efeitos das variações no ponto de referência.

O sistema deve ser projetado para que $T_{R_s} = T_R$

Em regime permanente $\varepsilon = 0$, pois: $\hat{T}_R = \hat{T}_m = \hat{T}$

$$\dot{q} = K_c \varepsilon + A \quad \Rightarrow \quad \dot{q} - \dot{q}_s = K_c \varepsilon$$

$$\dot{q}_s = K_c 0 + A \quad \dot{Q} = K_c \varepsilon$$

Transformada de Laplace: $\dot{Q}(s) = K_c \varepsilon(s)$

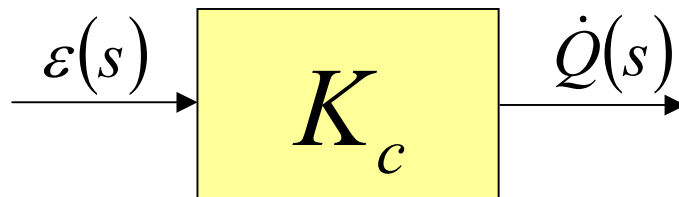
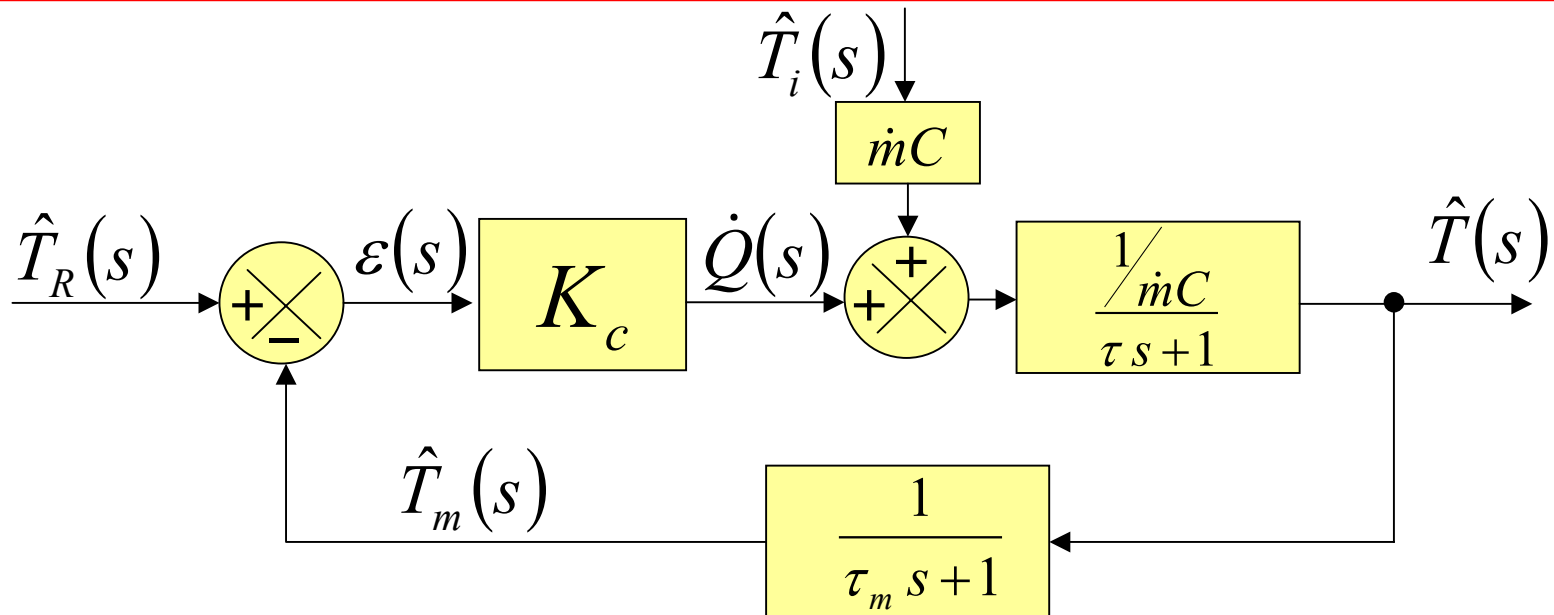
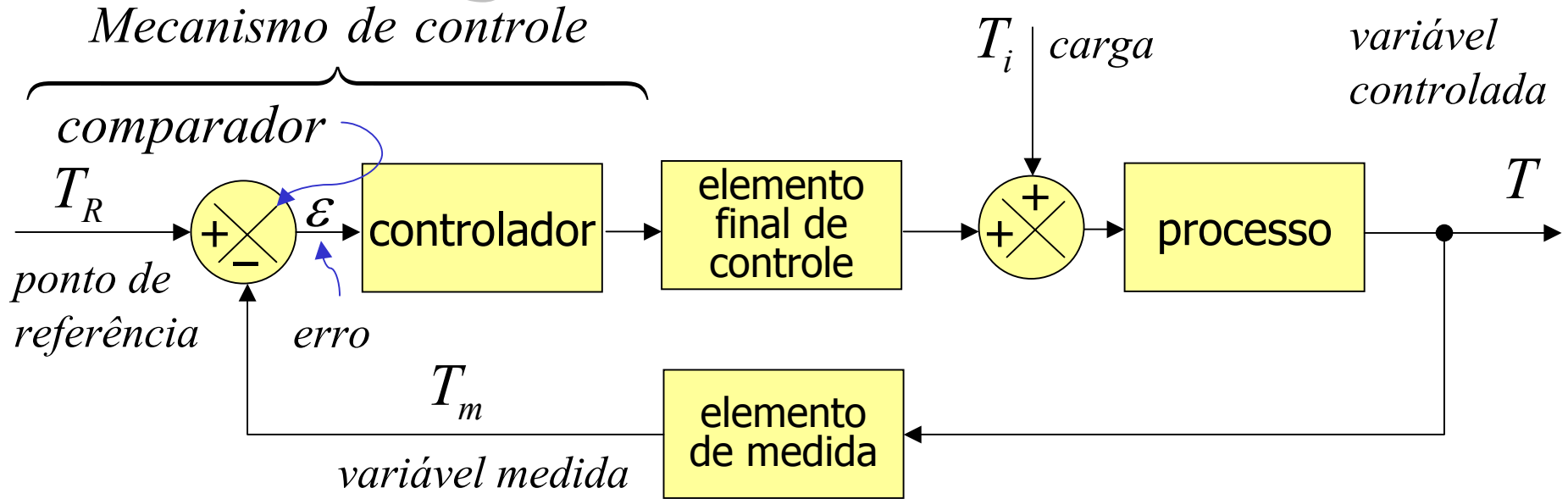
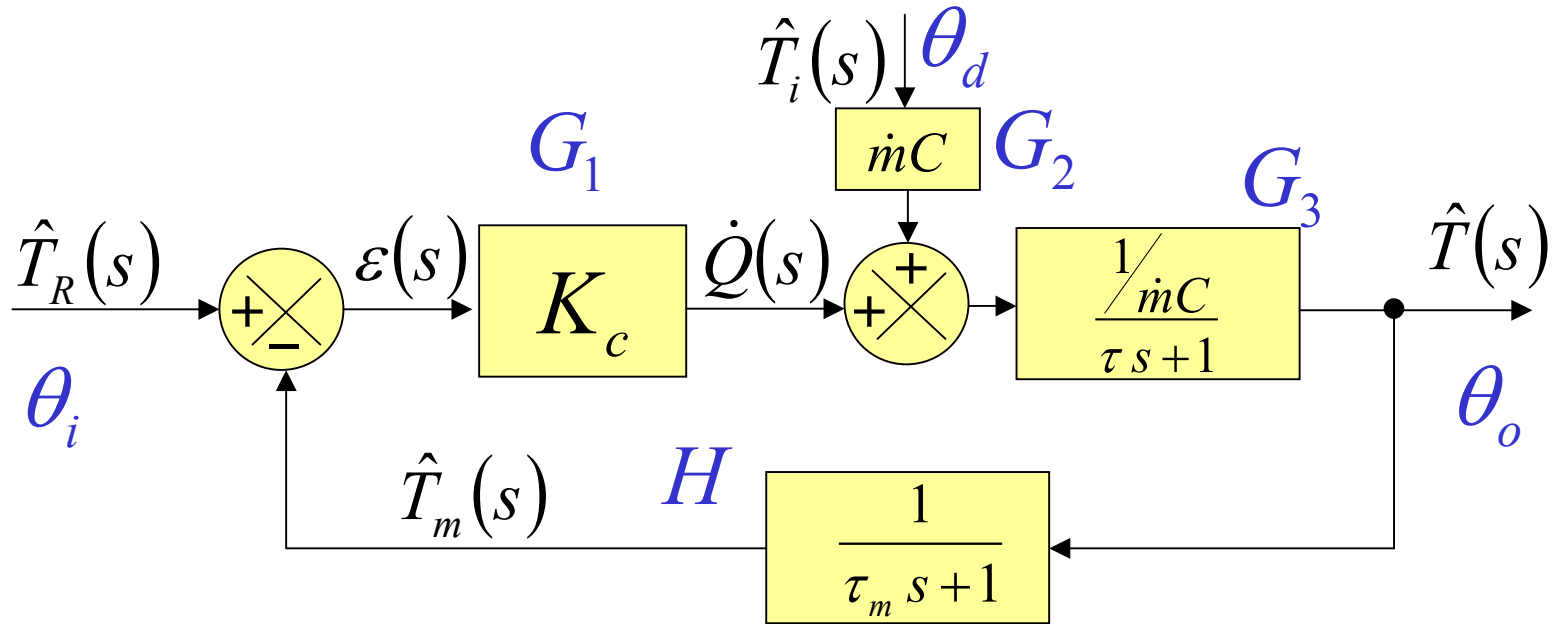


Diagrama de Blocos



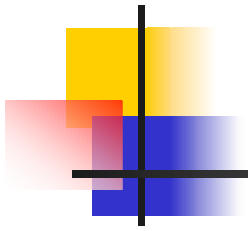
Função de transferência do sistema



$$[(\theta_i - H\theta_o)G_1 + \theta_d G_2]G_3 = \theta_o$$

$$\theta_o = \frac{G_1 G_3}{1 + H G_1 G_3} \theta_i + \frac{G_2 G_3}{1 + H G_1 G_3} \theta_d$$

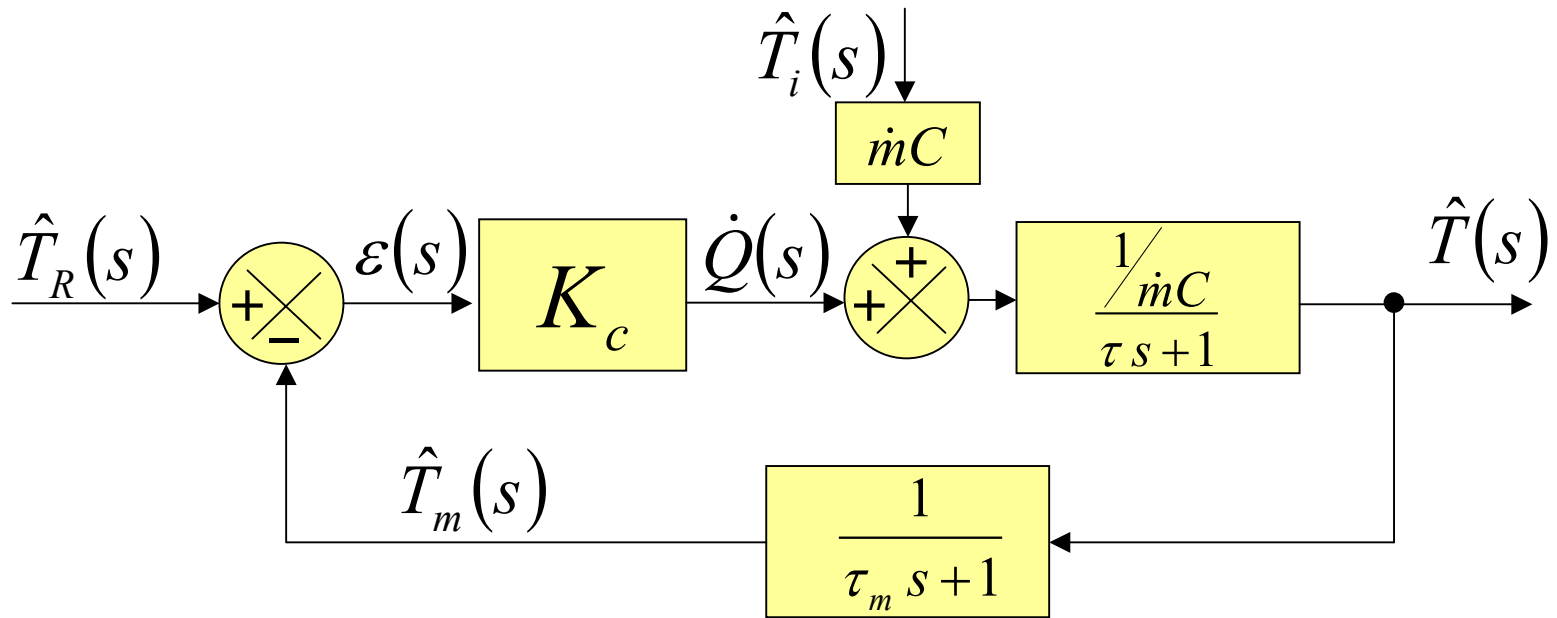
$$\hat{T}(s) = \frac{K_c \frac{1}{mC}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1}{mC}} \hat{T}_R(s) + \frac{mC \frac{1}{mC}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1}{mC}} \hat{T}_i(s)$$



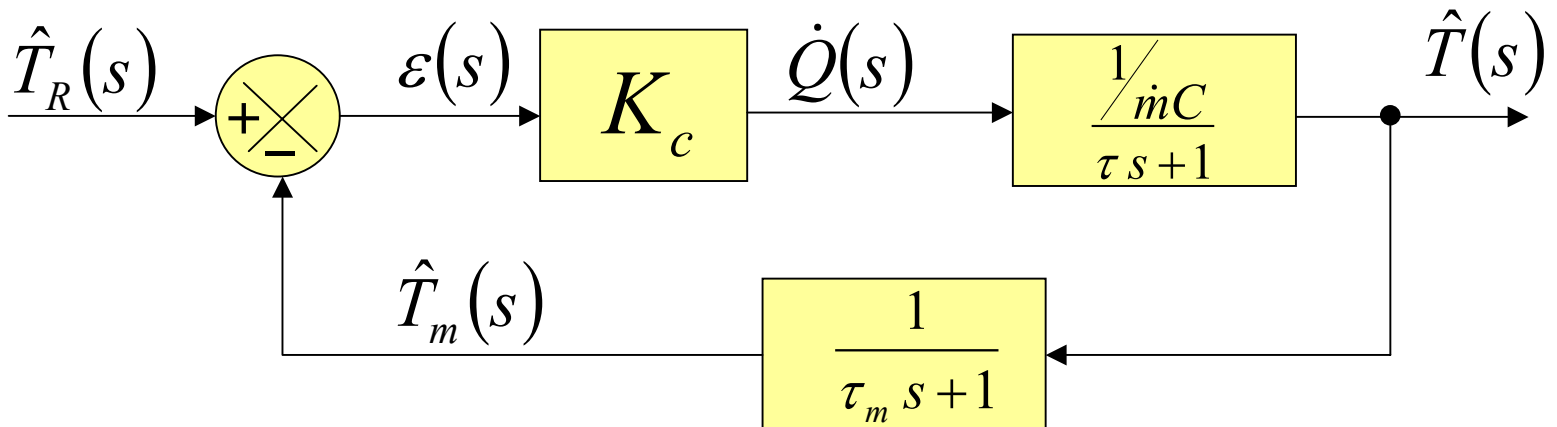
$$\hat{T}(s) = \frac{K_c \frac{1/\dot{m}C}{\tau s + 1}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1/\dot{m}C}{\tau s + 1}} \hat{T}_R(s) + \frac{\dot{m}C \frac{1/\dot{m}C}{\tau s + 1}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1/\dot{m}C}{\tau s + 1}} \hat{T}_i(s)$$

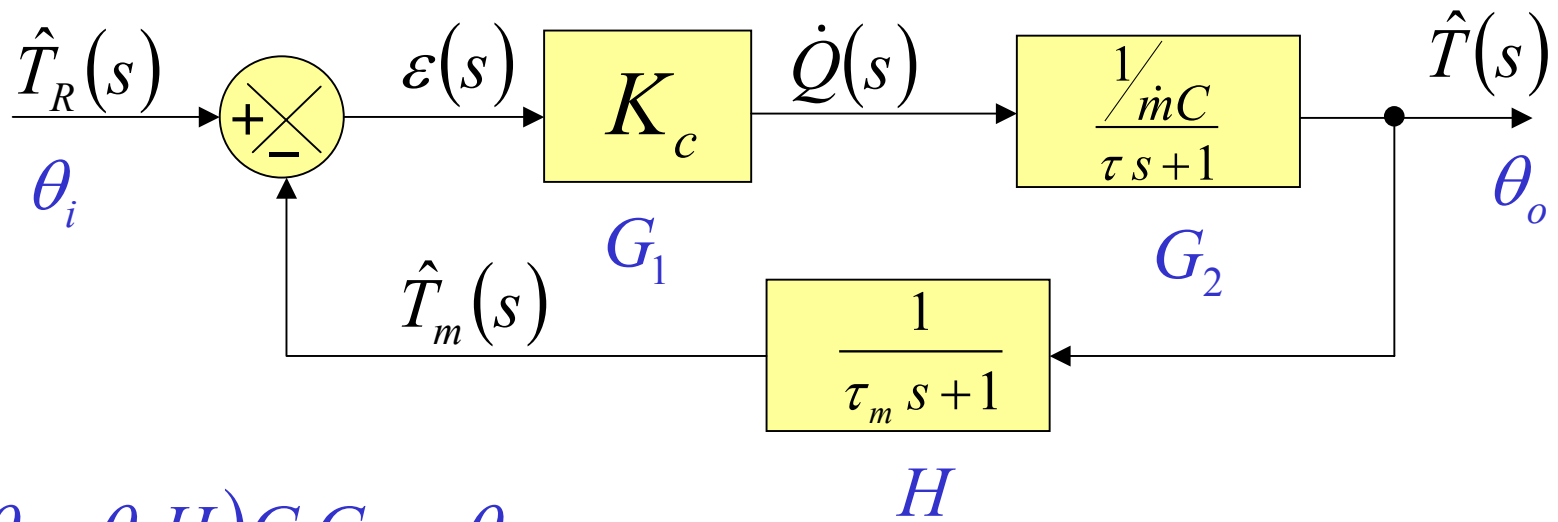
$$\hat{T}(s) = \frac{\frac{K_c}{\dot{m}C} (\tau_m s + 1)}{(\tau_m s + 1)(\tau s + 1) + \frac{K_c}{\dot{m}C}} \hat{T}_R(s) + \frac{(\tau_m s + 1)}{(\tau_m s + 1)(\tau s + 1) + \frac{K_c}{\dot{m}C}} \hat{T}_i(s)$$

Função de transferência do sistema



Para uma variação em $\hat{T}_R(s)$ e $\hat{T}_i(s) = 0$





$$(\theta_i - \theta_o H) G_1 G_2 = \theta_o$$

$$\frac{\theta_o}{\theta_i} = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

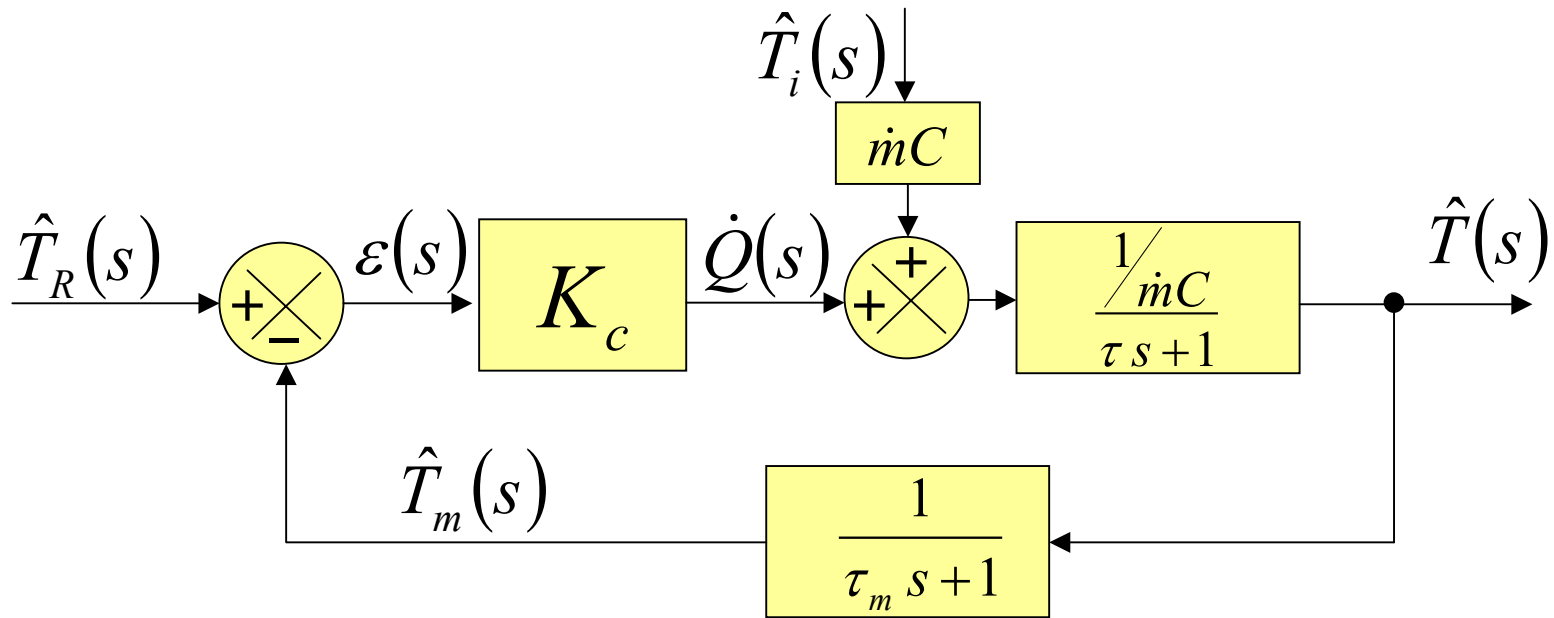
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{K_c \cdot \frac{1/\dot{m}C}{\tau s + 1}}{1 + K_c \cdot \frac{1/\dot{m}C}{\tau s + 1} \cdot \frac{1}{\tau_m s + 1}}$$

$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C} (\tau_m s + 1)}{(\tau s + 1)(\tau_m s + 1) + \frac{K_c}{\dot{m}C}}$$

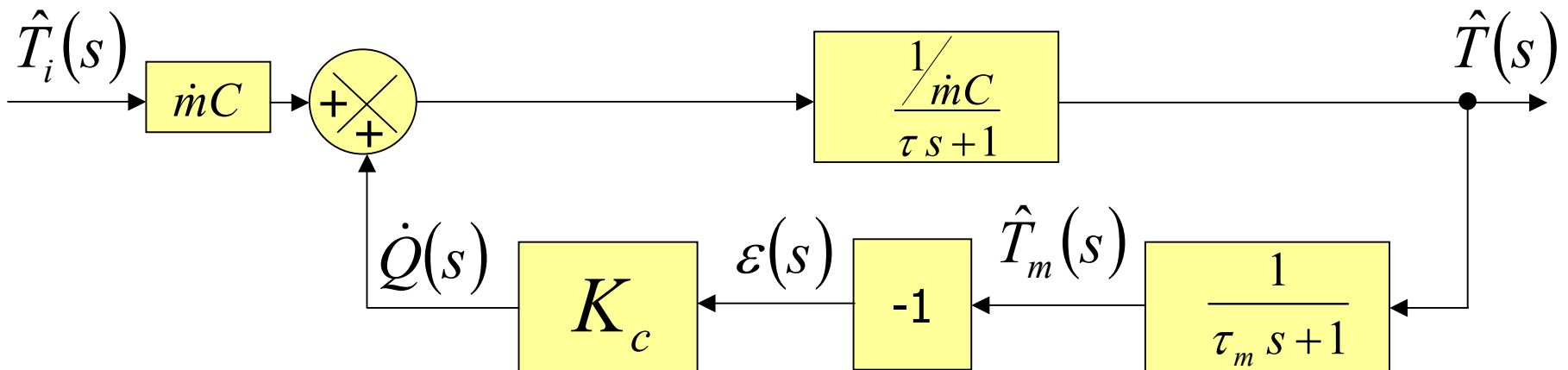
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{(\tau s + 1)(\tau_m s + 1) + \frac{K_c}{\dot{m}C}}$$

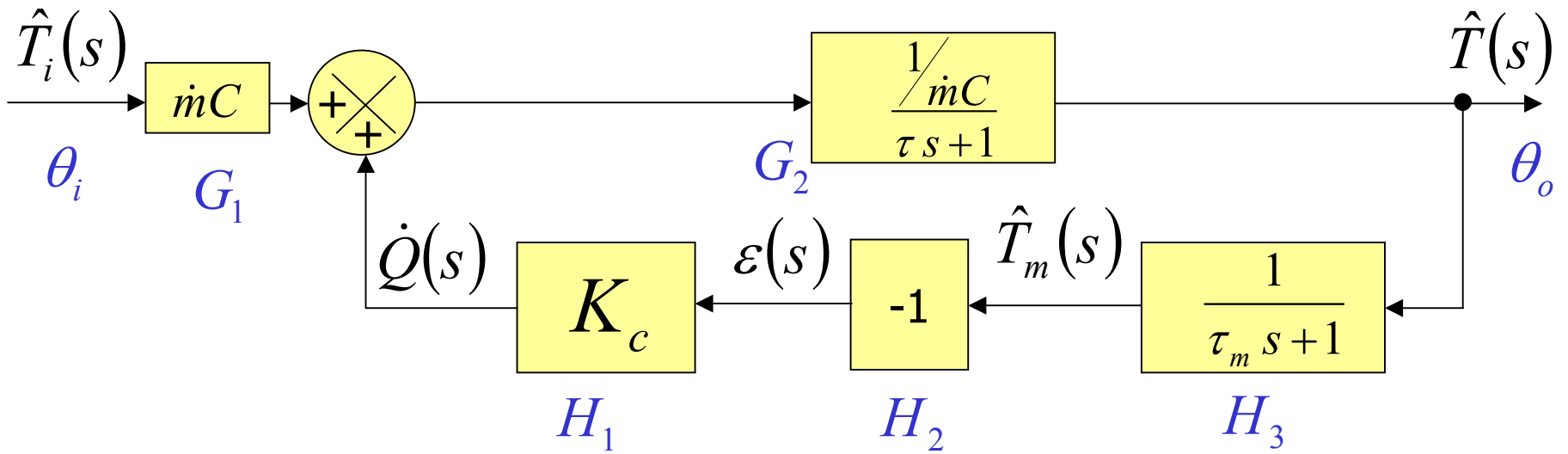
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

Função de transferência do sistema



Para uma variação em $\hat{T}_i(s)$ e $\hat{T}_R(s) = 0$





$$(\theta_i G_1 + \theta_o H_1 H_2 H_3) G_2 = \theta_o \quad \frac{\theta_o}{\theta_i} = \frac{G_1 G_2}{1 - G_2 H_1 H_2 H_3}$$

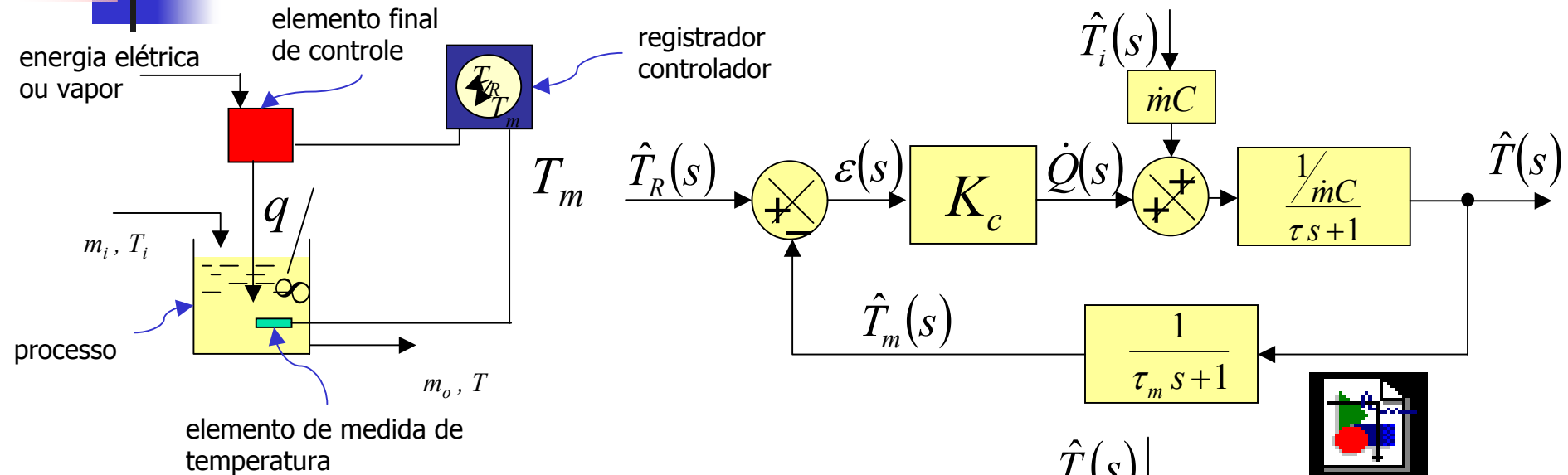
$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\frac{1}{\tau s + 1}}{1 + \frac{1/\dot{m}C}{\tau s + 1} \cdot K_c \cdot \frac{1}{\tau_m s + 1}}$$

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\frac{1}{\tau s + 1}}{1 + \frac{1/\dot{m}C}{\tau s + 1} \cdot K_c \cdot \frac{1}{\tau_m s + 1}}$$

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{(\tau s + 1)(\tau_m s + 1) + \frac{K_c}{\dot{m}C}}$$

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

Exemplo de solução com MATLAB Simulink



Dados:

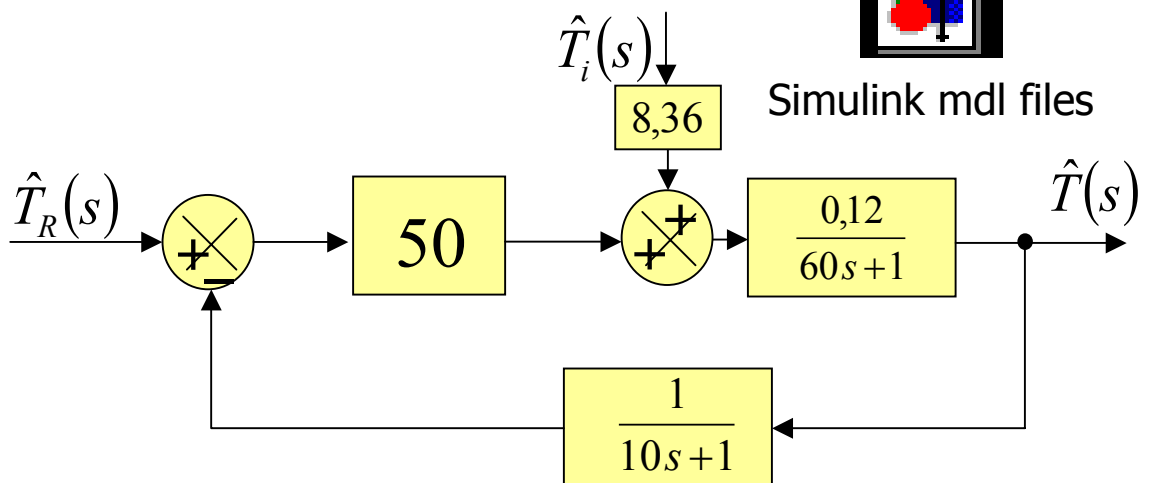
$$\tau = 60 \text{ s}$$

$$\dot{m} = 2,0 \text{ kg/s}$$

$$\tau_m = 10 \text{ s}$$

$$C = 4,18 \text{ kJ/kg} \cdot ^\circ\text{C}$$

$$K_c = 50 \text{ kW/}^\circ\text{C}$$



Simulink mdl files

Para uma entrada Degrau unitário em T_R , qual o erro em regime permanente?

🔧 Função de transferência:

🔧 Entrada degrau unitário em T_R :

$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

$$\hat{T}_R(s) = \frac{1}{s}$$

$$\hat{T}(s) = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)} \frac{1}{s}$$

✓ Teorema do valor final:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$



Valor final:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

$$\lim_{t \rightarrow \infty} T(t) = \lim_{s \rightarrow 0} s \cdot \frac{\frac{K_c}{\dot{m}C} (\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m) s + \left(\frac{K_c}{\dot{m}C} + 1 \right)} \frac{1}{s} \Rightarrow \lim_{t \rightarrow \infty} T(t) = \frac{\frac{K_c}{\dot{m}C}}{\frac{K_c}{\dot{m}C} + 1}$$



Erro:

$$Erro = T_R(\infty) - T(\infty)$$

$$Erro = 1 - \frac{\frac{K_c}{\dot{m}C}}{\frac{K_c}{\dot{m}C} + 1} \Rightarrow Erro = \frac{1}{\frac{K_c}{\dot{m}C} + 1}$$

Para uma entrada Degrau unitário em T_i , qual o erro em regime permanente, em relação à T_R ?

📌 Função de transferência:

📌 Entrada degrau unitário em T_i :

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

$$\hat{T}_i(s) = \frac{1}{s}$$

$$\hat{T}(s) = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)} \frac{1}{s}$$

✓ Teorema do valor final:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

 Valor final:

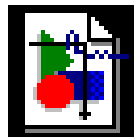
$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

$$\lim_{t \rightarrow \infty} T(t) = \lim_{s \rightarrow 0} s \cdot \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)} \frac{1}{s} \Rightarrow \lim_{t \rightarrow \infty} T(t) = \frac{1}{\frac{K_c}{\dot{m}C} + 1}$$

 Erro:

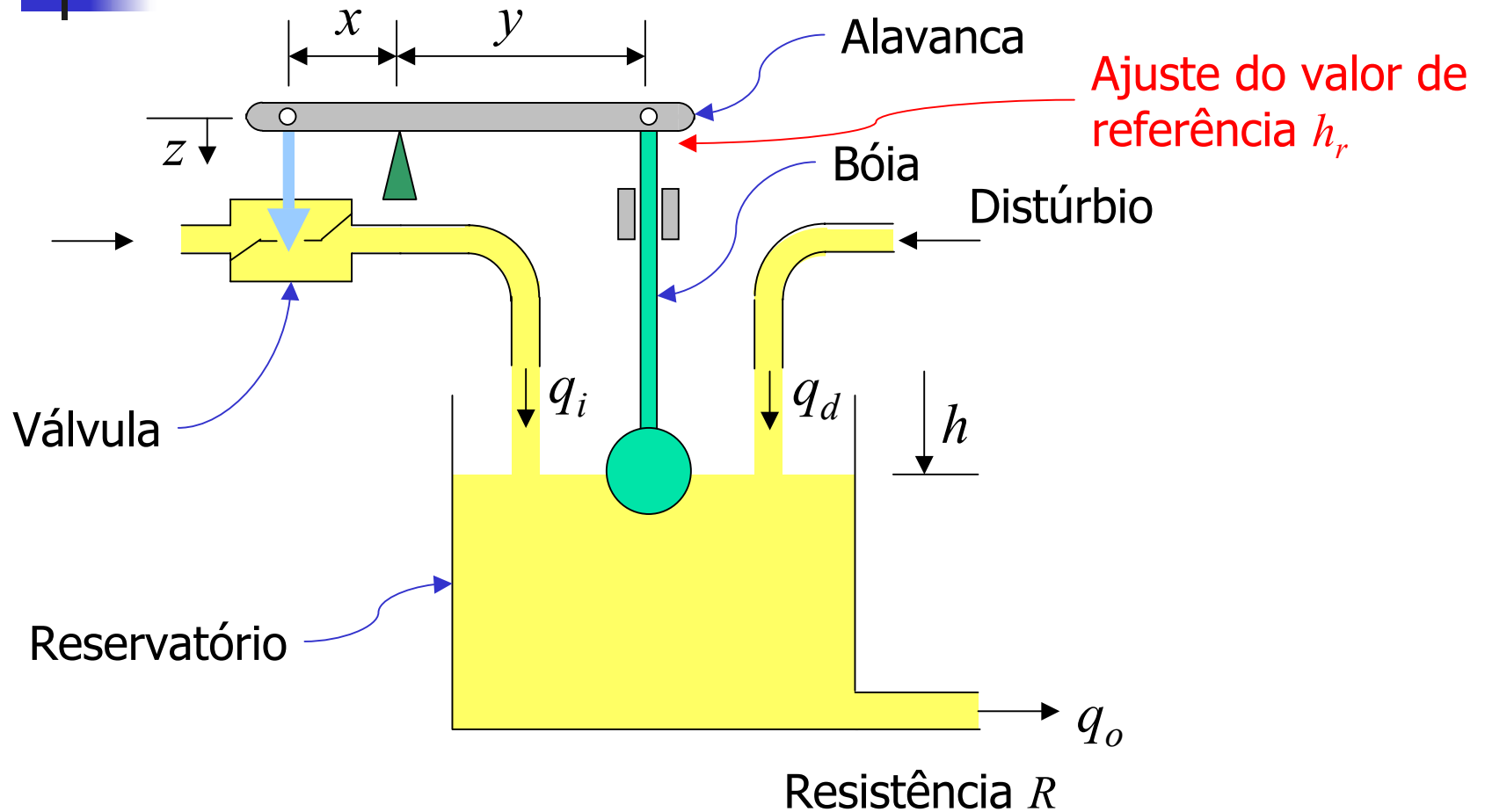
$$Erro = T_R(\infty) - T(\infty)$$

$$Erro = 0 - \frac{1}{\frac{K_c}{\dot{m}C} + 1} \Rightarrow Erro = \frac{-1}{\frac{K_c}{\dot{m}C} + 1}$$



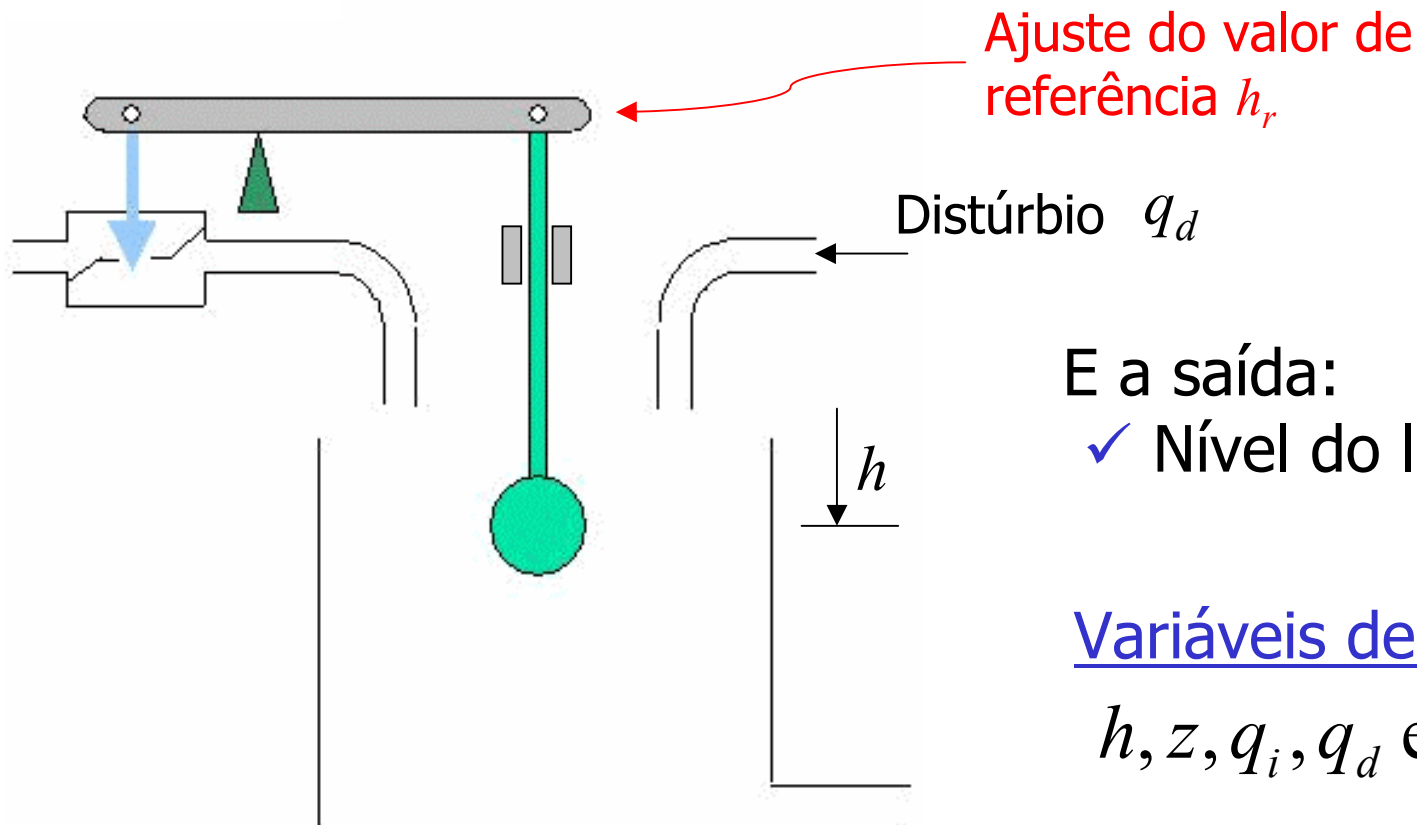
Simulink mdl files

Sistema de controle de nível



Determinar a relação entre as entradas:

- ✓ Valor de ajuste para o nível do líquido h_r
- ✓ Distúrbio para o nível de líquido q_d



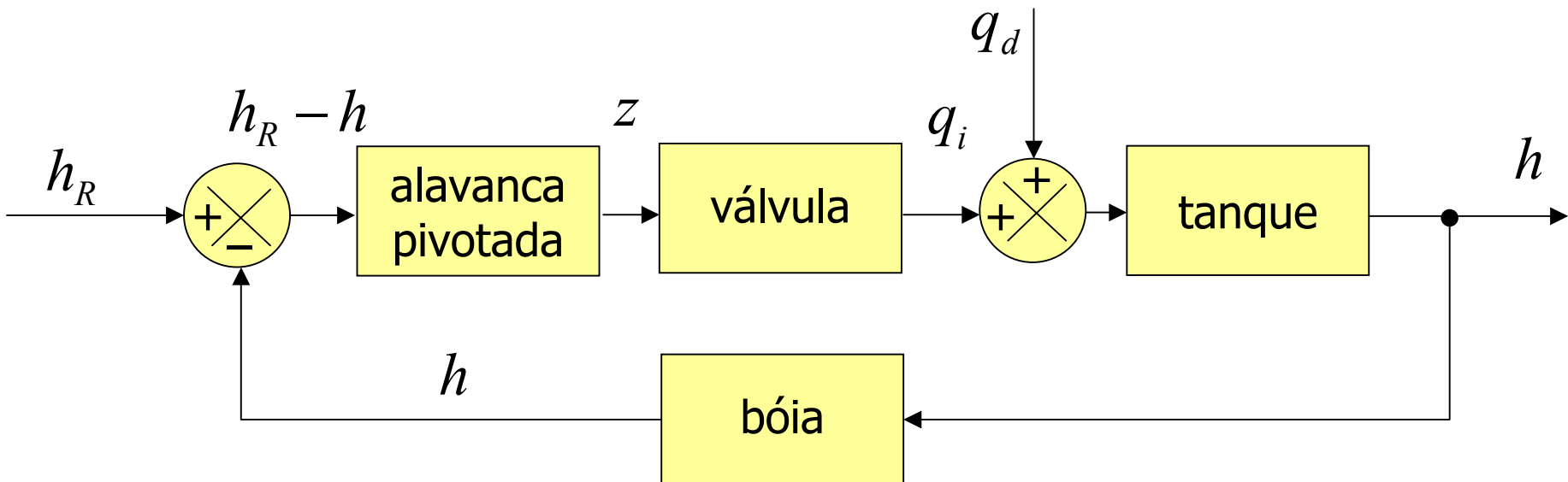
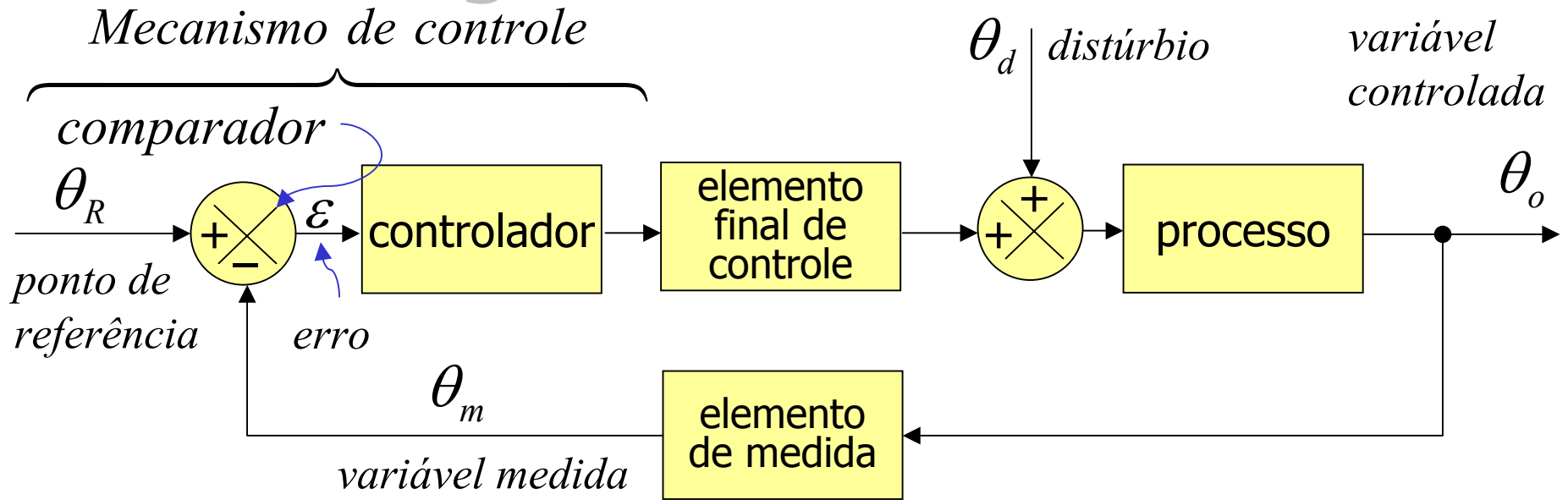
E a saída:

- ✓ Nível do líquido h

Variáveis desvio:

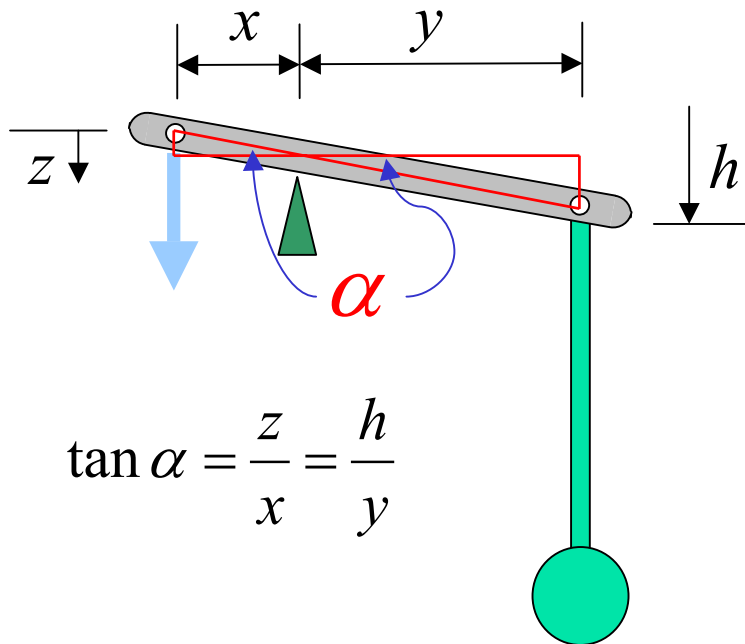
$$h, z, q_i, q_d \text{ e } q_o$$

Diagrama de Blocos



Desenvolvimento do Diagrama de Blocos

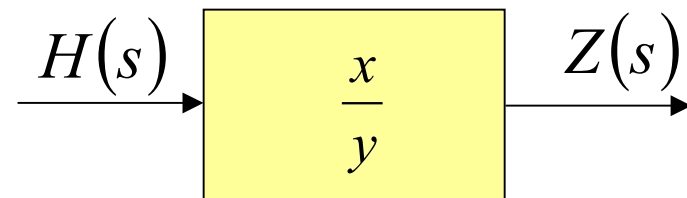
Alavanca pivotada



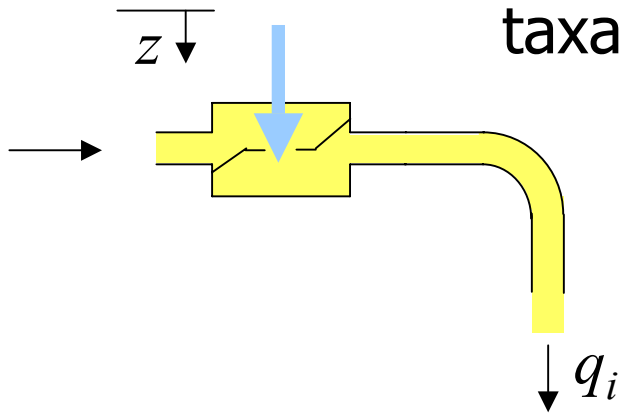
$$\frac{\text{Saída}}{\text{Entrada}} = \frac{\text{Distância do pivô na saída}}{\text{Distância do pivô na entrada}}$$

$$\frac{z}{h} = \frac{x}{y}$$

$$Z(s) = \frac{x}{y} H(s)$$



Válvula

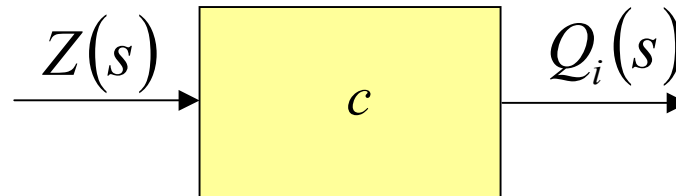


↪ A entrada z para a válvula determina a taxa de escoamento q_i na saída da válvula.

↪ A relação entre a entrada e a saída pode ser linearizada:

$$q_i = c z$$

$$Q_i(s) = c Z(s)$$



Tanque

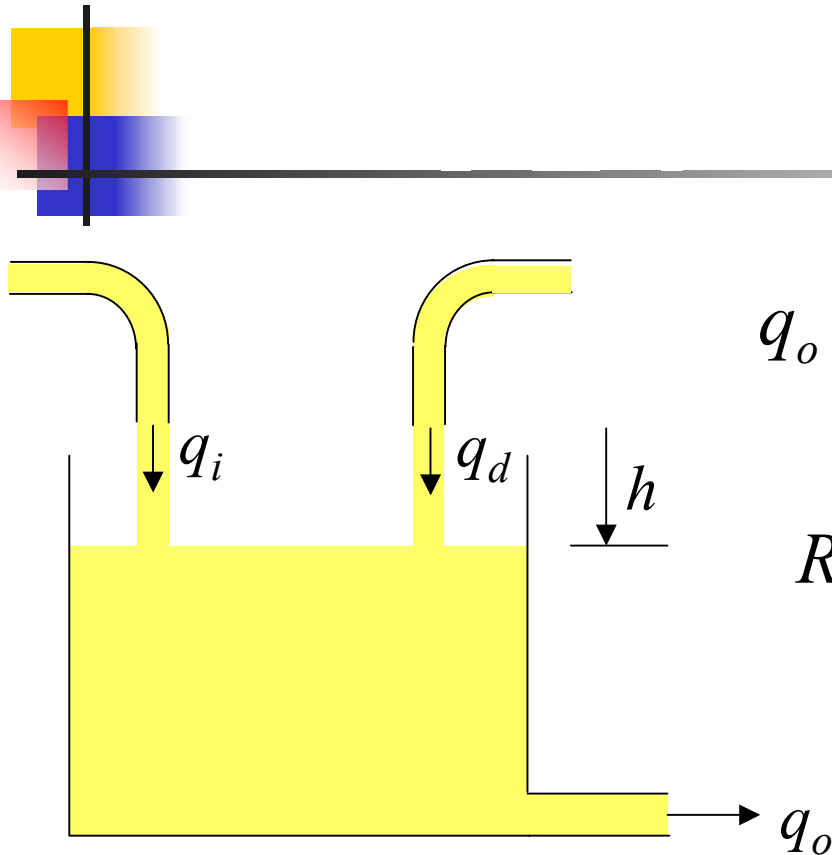
Conservação da massa:

$$q_i + q_d - q_o = A \frac{d h}{d t}$$

$$q_o = \frac{h}{R} \Rightarrow q_i + q_d = A \frac{d h}{d t} + \frac{h}{R}$$

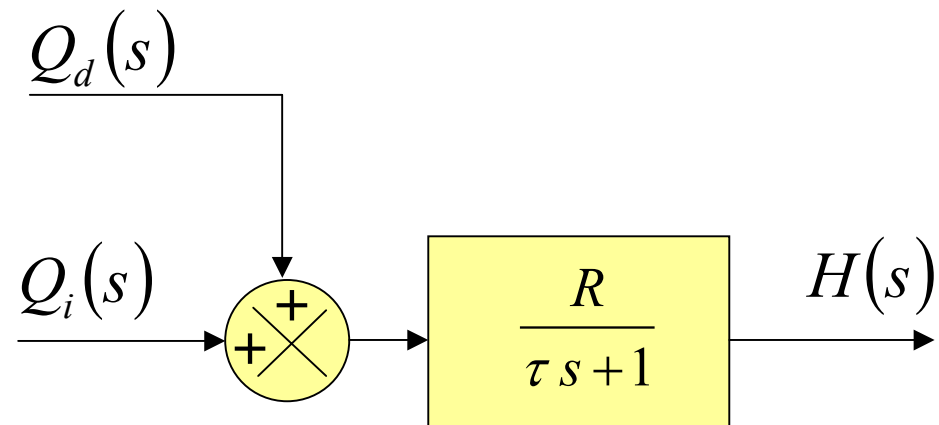
$$R[Q_i(s) + Q_d(s)] = ARs H(s) + H(s)$$

$$\tau = AR$$



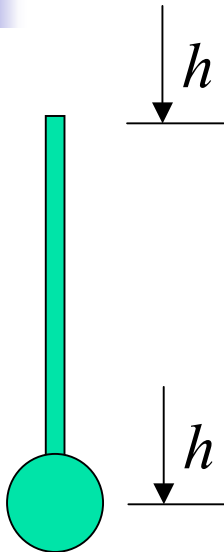
Resistência R

$$H(s) = \frac{R Q_i(s)}{\tau s + 1} + \frac{R Q_d(s)}{\tau s + 1}$$



Bóia

A realimentação é o movimento da bóia



- O movimento da bóia transmite diretamente o sinal de altura de líquido em movimento da alavanca.
- Portanto a realimentação é unitária.

$$H(s) = 1 \cdot H(s)$$

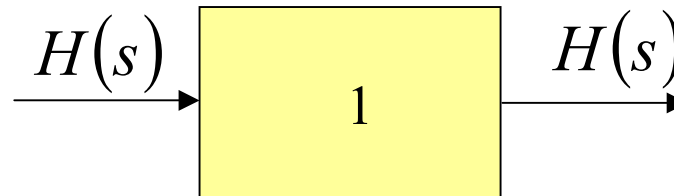
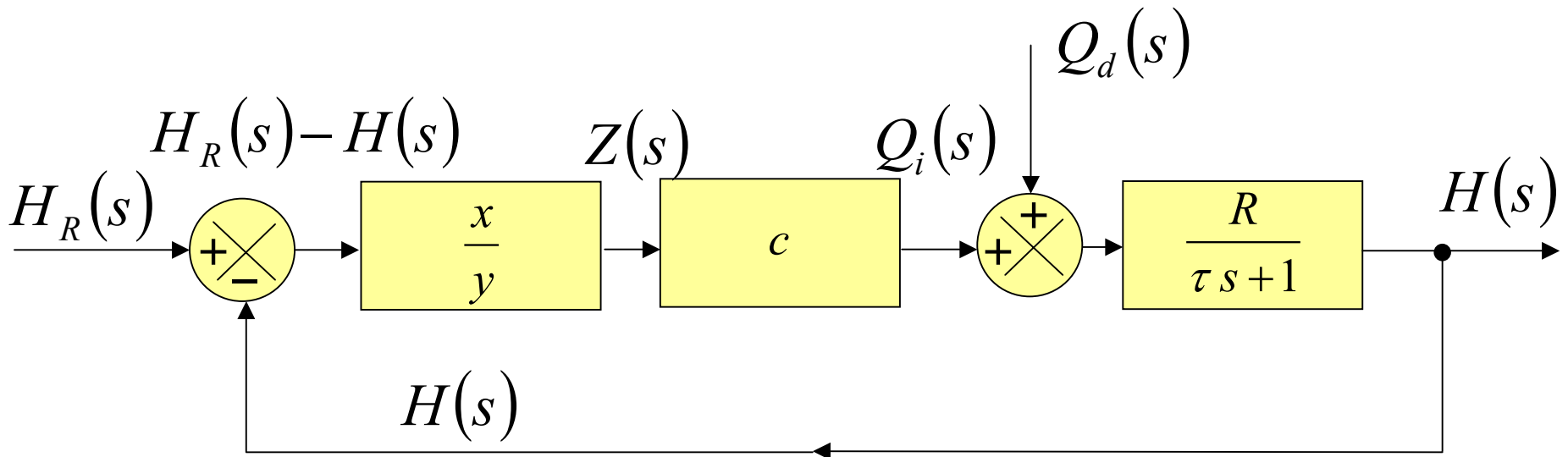
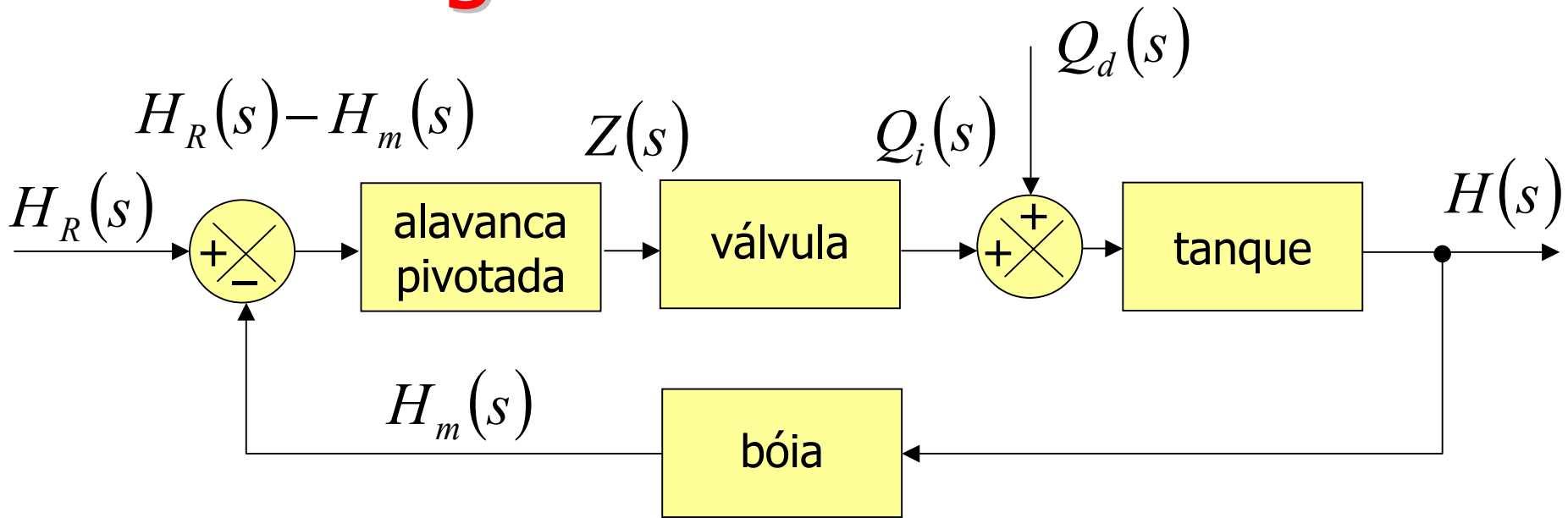
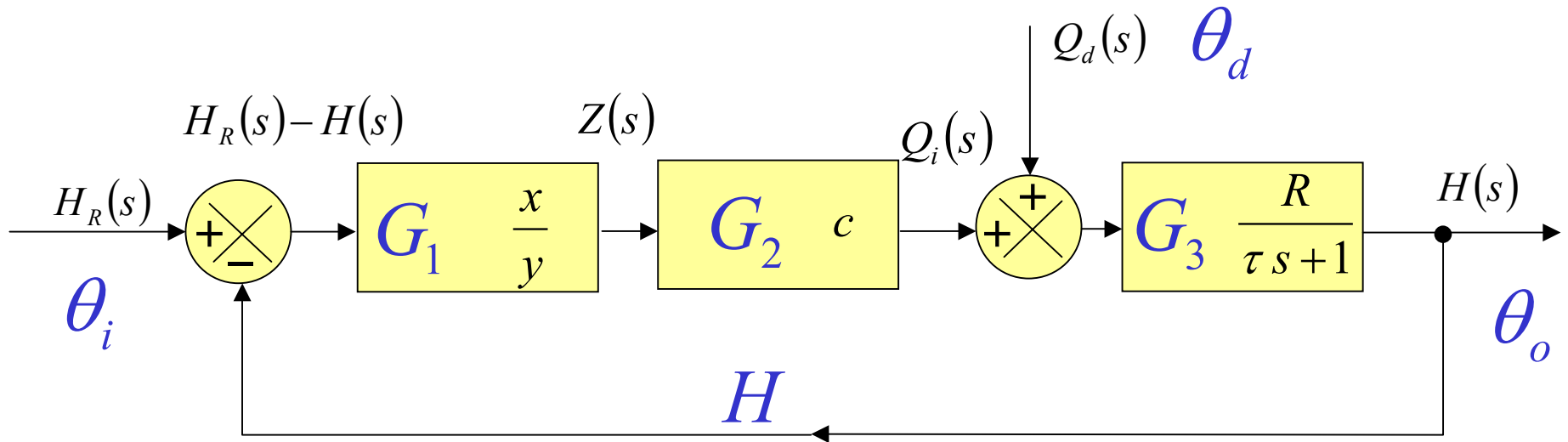


Diagrama de Blocos




Função de Transferência



$$[(\theta_i - \theta_o H) G_1 G_2 + \theta_d] G_3 = \theta_o \quad \theta_o = \frac{G_1 G_2 G_3 \theta_i + G_3 \theta_d}{1 + G_1 G_2 G_3 H}$$

$$H(s) = \frac{\frac{x}{y} c \frac{R}{\tau s + 1} H_R(s) + \frac{R}{\tau s + 1} Q_d(s)}{1 + \frac{x}{y} c \frac{R}{\tau s + 1}}$$



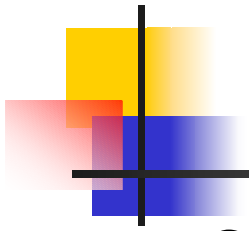
$$H(s) = \frac{\frac{x}{y} c \frac{R}{\tau s + 1} H_R(s) + \frac{R}{\tau s + 1} Q_d(s)}{1 + \frac{x}{y} c \frac{R}{\tau s + 1}} \Rightarrow H(s) = \frac{\frac{x}{y} c R H_R(s) + R Q_d(s)}{\tau s + 1 + \frac{x}{y} c R}$$

$$H(s) = \frac{\frac{x}{y} c R}{\tau s + 1 + \frac{x}{y} c R} H_R(s) + \frac{R}{\tau s + 1 + \frac{x}{y} c R} Q_d(s)$$

Definindo as constantes:

$$a = \frac{1 + \frac{x}{y} c R}{\tau} \quad K_1 = \frac{\frac{x}{y} c R}{1 + \frac{x}{y} c R} \quad K_2 = \frac{R}{1 + \frac{x}{y} c R}$$

$$H(s) = \frac{K_1 a}{s + a} H_R(s) + \frac{K_2 a}{s + a} Q_d(s)$$


$$H(s) = \frac{K_1 a}{s + a} H_R(s) + \frac{K_2}{s + a} Q_d(s)$$

Com uma entrada degrau unitário em $H_R(s)$, sem distúrbio:

$$H_R(s) = \frac{1}{s} \quad H(s) = \frac{K_1 a}{s(s + a)} \quad h(t) = K_1 (1 - e^{-at})$$

Com uma entrada degrau unitário em $Q_d(s)$, sem alterar o ponto de ajuste:

$$Q_d(s) = \frac{1}{s} \quad H(s) = \frac{K_2 a}{s(s + a)} \quad h(t) = K_2 (1 - e^{-at})$$

Dados:

$$A = 0,3 \text{ m}^2 \quad R = 300 \frac{\text{m}}{\text{m}^3/\text{s}}$$

$$\tau = 90 \text{ s}$$

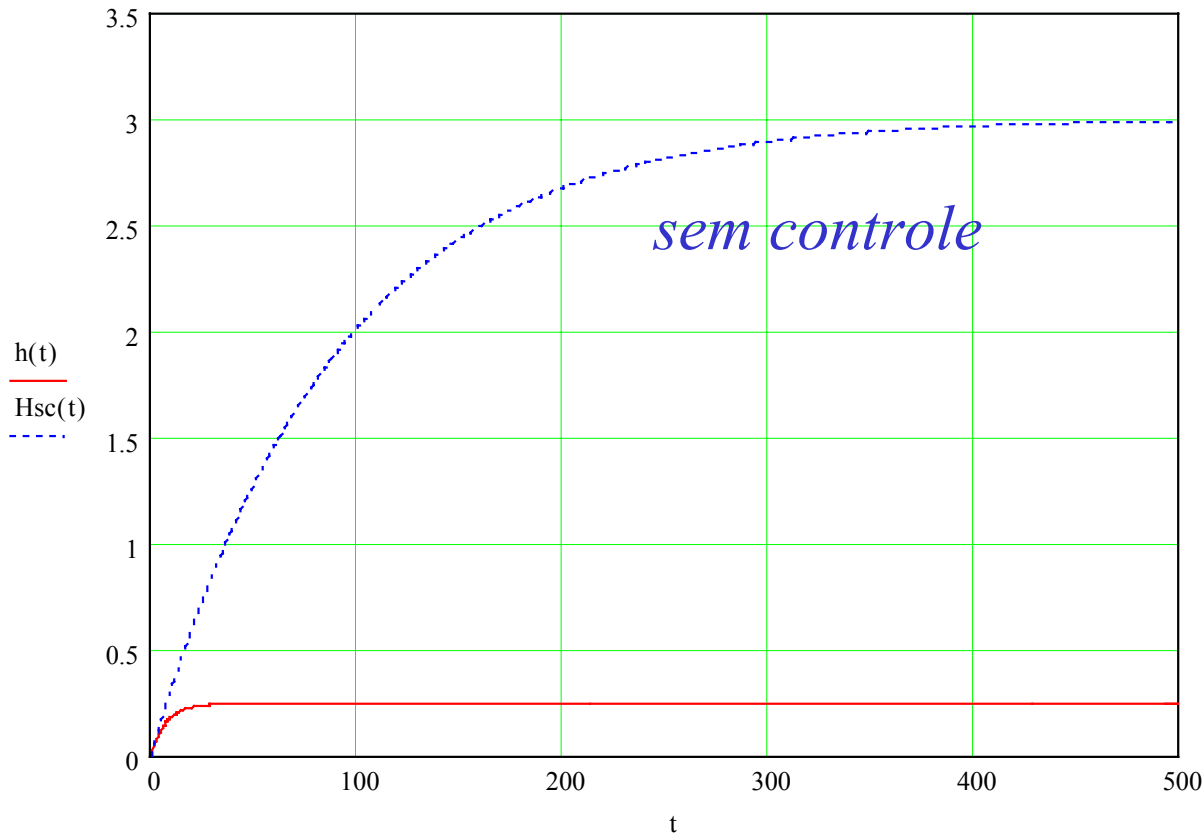
$$x = 0,3 \text{ m}$$

$$y = 1,2 \text{ m}$$

$$c = 0,15 \frac{\text{m}^2}{\text{s}}$$

distúrbio $Q_d = 0,01 \frac{\text{m}^3}{\text{s}}$

$$h(t) = K_2 (1 - e^{-at})$$



$$h(t) = 3 (1 - e^{-0,011.t})$$



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$$h(t) = 0,25 (1 - e^{-0,136.t})$$