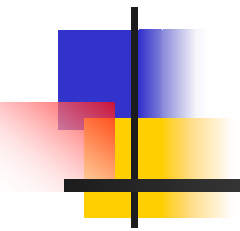
EQUAÇÕES DIFERENCIAIS



Origem:

- Primitivas
- Problemas geométricos
- Fenômenos físicos

FENÔMENOS FÍSICOS

MODELOS DE SISTEMAS

Modelos matemáticos

Leis físicas fundamentais

- Equações de conservação
- Equações constitutivas

Sistemas

- Mecânicos
- Elétricos
- Térmicos
- Fluidos

Equações de Conservação

Conservação da quantidade de movimento linear

$$\sum F = \frac{d}{dt}[mv] \qquad \text{para } m\text{=const.} \qquad \sum F - m\frac{dv}{dt} = 0$$

$$\sum F = m.a$$

Conservação da quantidade de movimento angular

$$\sum T = \frac{d}{dt}[J\omega] \qquad \sum T - J\frac{d\omega}{dt} = 0$$

$$\sum T = J.\alpha$$

Equações de Conservação

Conservação da carga elétrica (Lei de Kirchoff)

$$\sum i_{nó} = \frac{dQ}{dt} = C\frac{de}{dt}$$

$$\sum i_{nó} - C\frac{de}{dt} = 0$$

$$\sum i_{nó} = C\frac{d\,e}{dt}$$

Conservação da massa

$$\sum \dot{m} = \frac{d}{dt}[\rho V] = \rho\dot{V} + V\dot{\rho}$$

$$\sum \dot{m} - \frac{d}{dt}[\rho V] = 0$$

$$\sum \dot{m} = \frac{d\,m}{dt}$$

Equações de Conservação

Conservação da energia

$$\frac{d}{dt}\left[mu+\frac{mV^2}{2}+mz\right]_{vc}=\dot{Q}_{vc}-\dot{W}_{vc}+\sum\dot{m}_e\left(h_e+\frac{V_e^2}{2}+gZ_e\right)-\sum\dot{m}_s\left(h_s+\frac{V_s^2}{2}+gZ_s\right)$$

$$\frac{d\,mu}{dt}_{vc}=\dot{Q}_{vc}-\dot{W}_{vc}+\sum\dot{m}_e h_e-\sum\dot{m}_s h_s$$

$$m\,C_p\frac{d\,T}{dt}_{vc}=\dot{Q}_{vc}-\dot{W}_{vc}+\sum\dot{m}_e C_{p_e}\left(T_e-T_R\right)-\sum\dot{m}_s C_{p_s}\left(T_s-T_R\right)$$

Equações Constitutivas

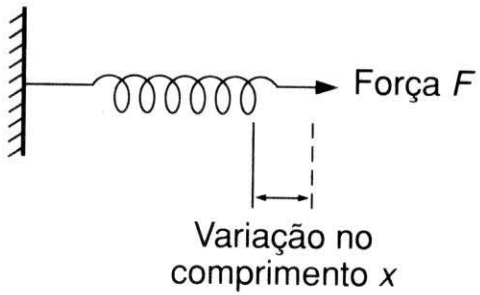
Leis físicas fundamentais que regem o comportamento de um elemento de um sistema

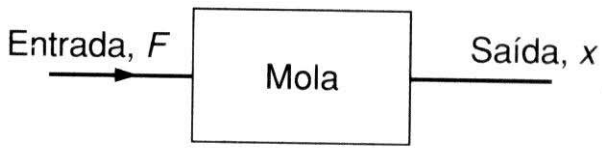
Exemplos

- Massa
- Mola
- Amortecedor

- Resistor
- Capacitor
- Indutor

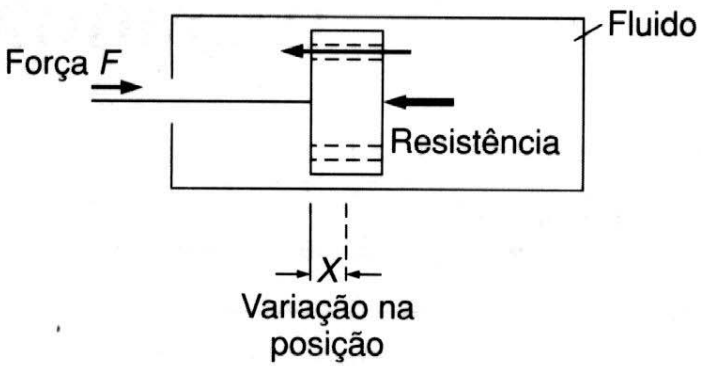
Sistemas Mecânicos — Mola

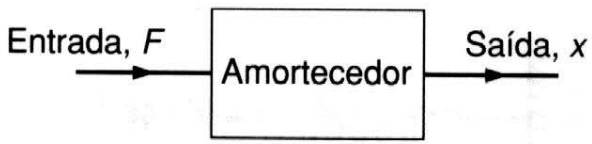




$$F = k\,x$$

Sistemas Mecânicos — Amortecedor

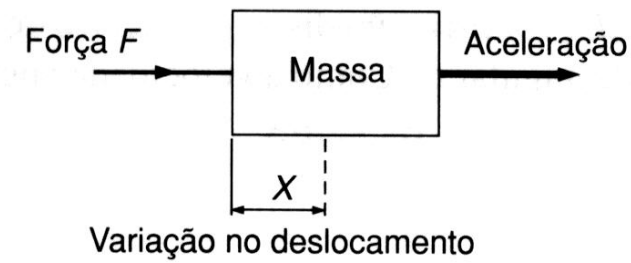




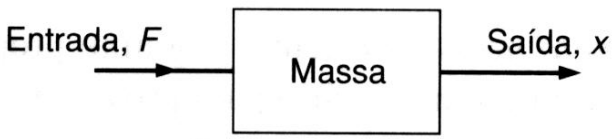
$$F = c\,v$$

$$F = c\frac{dx}{dt}$$

Sistemas Mecânicos — Massa



$$F = m\,a$$



$$F = m\,a = m\frac{dv}{dt} = m\frac{d\left(dx/dt\right)}{dt} = m\frac{d^2x}{dt^2}$$

Sistemas Mecânicos em Rotação

Mola torcional

$$T = k\theta$$

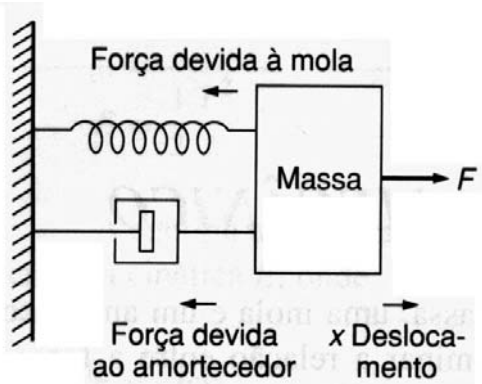
Amortecedor rotativo

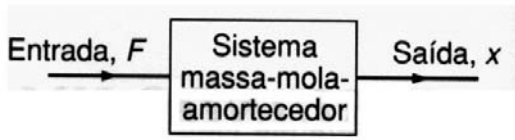
$$T = c\omega = c\frac{d\theta}{dt}$$

Inércia

$$T = J\alpha = J\frac{d\omega}{dt} = J\frac{d\left(d\theta/dt\right)}{dt} = J\frac{d^2\theta}{dt^2}$$

Construindo um Modelo para um Sistema Mecânico





Conservação da quantidade de movimento linear

$$\sum F_e = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

Somatório de forças aplicadas à massa *m*

$$\sum F_e = F - F_{mola} - F_{amortecedor}$$

$$\sum F_e = F - kx - cv$$

$$F - kx - cv = m\frac{d^2x}{dt^2} \qquad m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F$$

Freqüência angular natural

$$\omega_n = \sqrt{\frac{k}{m}}$$

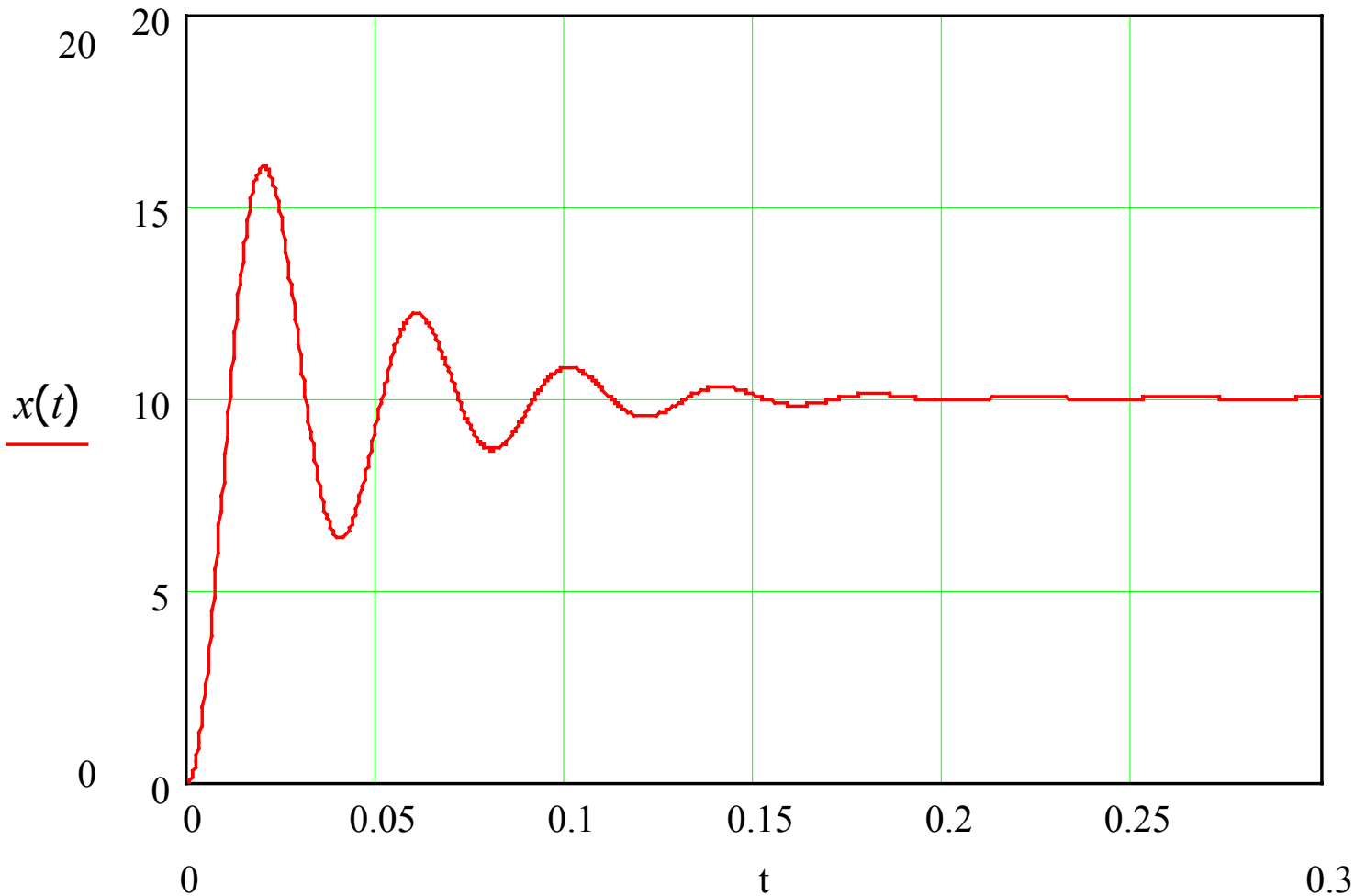
Razão de amortecimento

$$\zeta = \frac{c}{2\sqrt{(mk)}}$$

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dx}{dt} + x = \frac{F}{k}$$

Resposta de um sistema de 2ª ordem

$$x(t) = \frac{F}{k}\left[1 - e^{(-25,28)t}\left[\cos 156t + 0,162 \operatorname{sen} 156t\right]\right]$$



Edita gráfico

Mathcad Document

Sistemas Mecânicos de Rotação

$$T - k\theta - c\omega = J\frac{d^2\theta}{dt^2} \qquad J\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + k\theta = T$$

Freqüência angular natural $\omega_n = \sqrt{\frac{k}{J}}$

Razão de amortecimento $\zeta = \frac{c}{2\sqrt{(Jk)}}$

$$\frac{1}{\omega_n^2}\frac{d^2\theta}{dt^2} + \frac{2\zeta}{\omega_n}\frac{d\theta}{dt} + \theta = \frac{T}{k}$$

Exemplo de Sistema Mecânico

Suspensão de automóvel

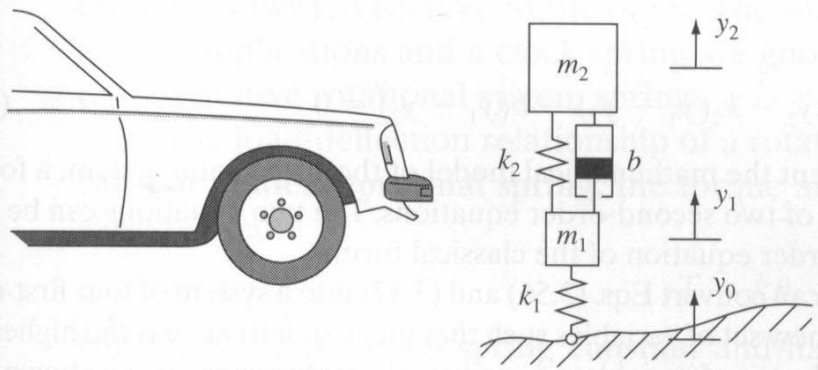


Figure 3.15 Auto front end and model.

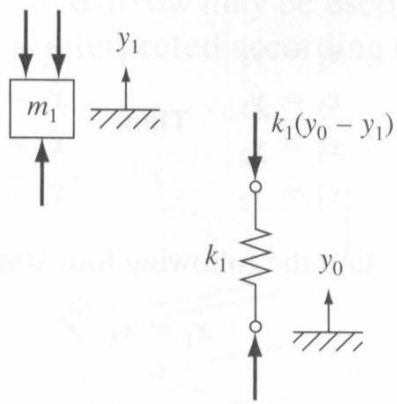
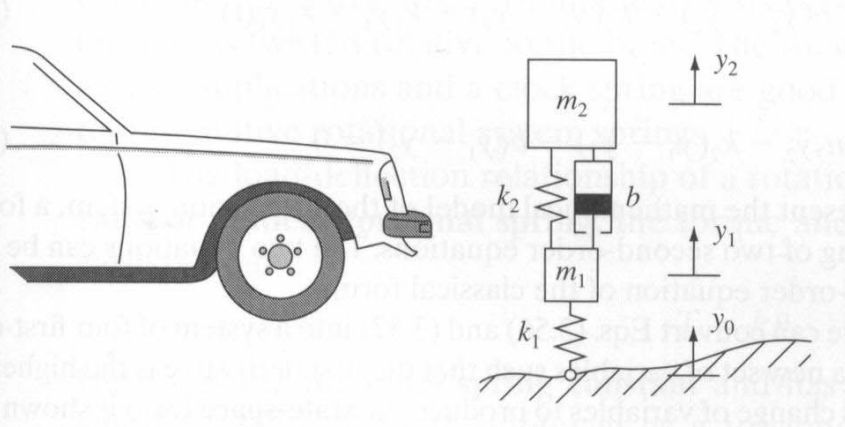
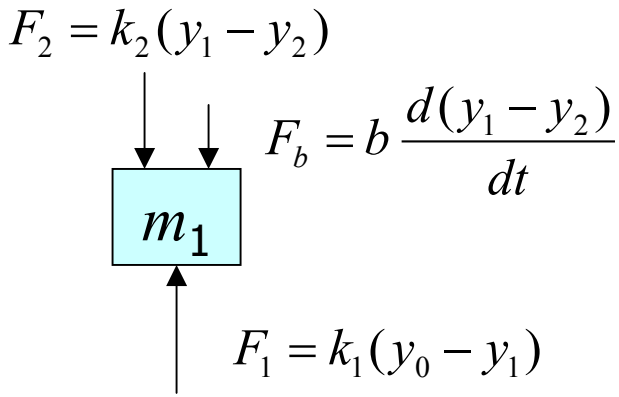


Figure 3.16 Free-body diagram of m_1.

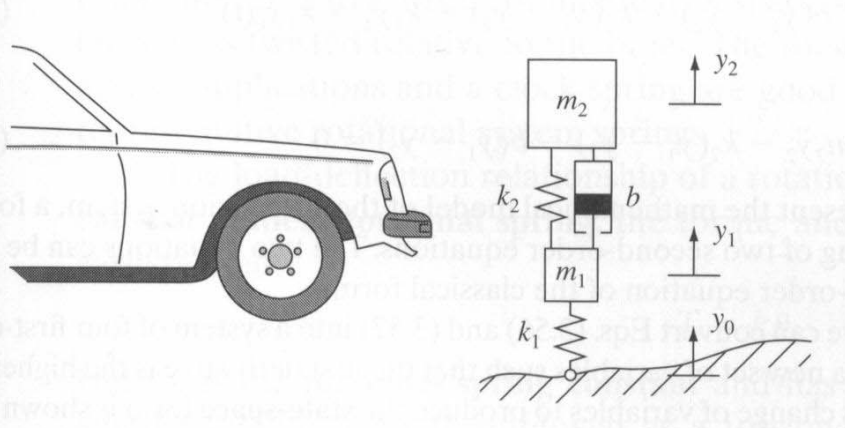


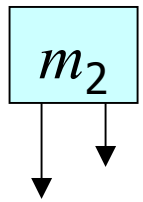


$$m_1 a = \sum F \qquad m_1 \frac{d^2 y_1}{dt^2} = \sum F$$

$$m_1 \frac{d^2 y_1}{dt^2} = F_1 - F_2 - F_b$$

$$m_1 \frac{d^2 y_1}{dt^2} + b\frac{d(y_1 - y_2)}{dt} + k_2(y_1 - y_2) = k_1(y_0 - y_1)$$





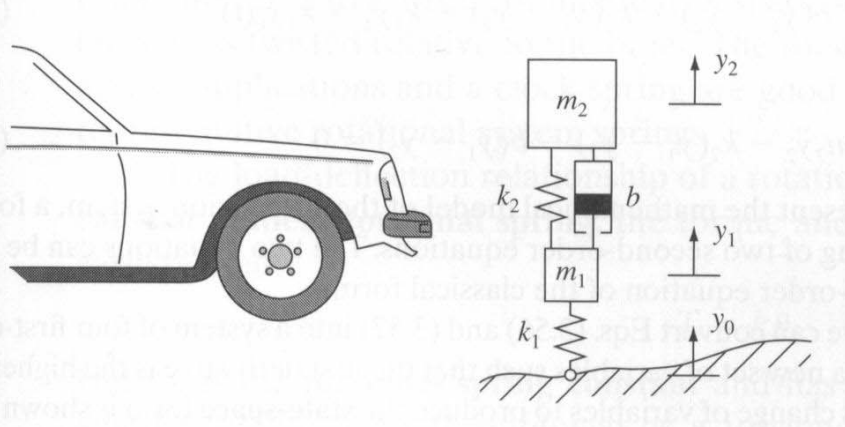
$$F_b = b\frac{d(y_2 - y_1)}{dt}$$

$$F_2 = k_2(y_2 - y_1)$$

$$m_2 a = \sum F \qquad m_2 \frac{d^2 y_2}{dt^2} = \sum F$$

$$m_2 \frac{d^2 y_2}{dt^2} = -F_2 - F_b$$

$$m_2 \frac{d^2 y_2}{dt^2} + b\frac{d(y_2 - y_1)}{dt} + k_2(y_2 - y_1) = 0$$



$$m_1 \frac{d^2 y_1}{dt^2} + b \frac{d(y_1 - y_2)}{dt} + k_2 (y_1 - y_2) = k_1 (y_0 - y_1)$$

$$m_2 \frac{d^2 y_2}{dt^2} + b \frac{d(y_2 - y_1)}{dt} + k_2 (y_2 - y_1) = 0$$

Exemplo de Sistema Mecânico

Suspensão de automóvel

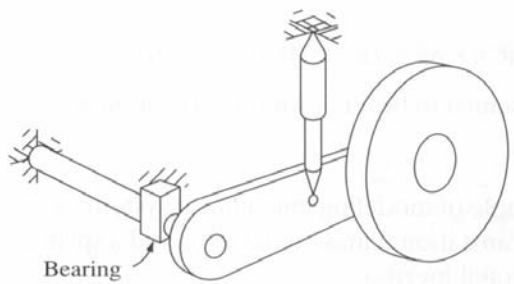
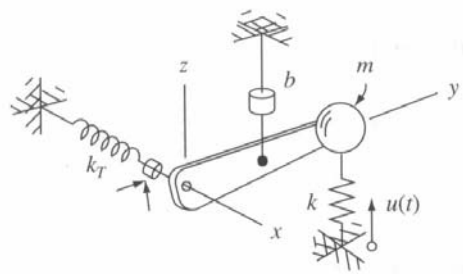


Figure 3.27 Automotive suspension system.



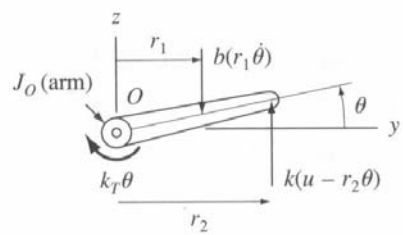


Figure 3.28 Suspension schematic and free-body diagram.